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THEORY OF SOLID AND BRACED ELASTIC ARCHES,

APPLIED TO ARCH BRIDGES AND ROOFS IN IRON, WOOD, CONCRETE OR OTHER MATERIAL. GRAPHICAL ANALYSIS.

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II.

APPLICATION TO VOUSSOIR ARCHES.

35. It has been previously observed (art. 1) that, in certain cases, the treatment of the voussoir arch falls under that pertaining to the solid arch. In fact, conceive a *solid* stone arch in which the pressure curve keeps within such limits that only the resistance to compression of the stone is brought into play. Now, if this solid arch is divided up into any number of voussoirs, the joints being perfectly cut, the conditions are unchanged; since the change of direction, Δa of tangents, at ends of a portion of the arch whose length is s, is the same in either case, this change being due entirely to the *compression*, on any supposed joint, not being uniform, but uniformly increasing in going towards one edge. The values $\overline{cd}=y.\Delta a$, $de=x.\Delta a$, of art. 12, are thus the same in either case, as well as the expressions for the total horizontal and vertical displacements following.

It should be noted, however, that in the value for Δa, given by eq. 2, art. 9, that the uniform thrusts T, on the cross sections, which shorten the entire arch, are neglected. Its effect on the bending is very slight, and is included, further on, under temperature strains, and we may assume, that for the voussoir arch, eq. 2, represents the true value of Δa, and that I has the same value as for the solid arch. It must not be supposed by the last hypothesis that the voussoir arch can supply tensile resistances; we only use eq. 2, with the value of I as for a solid arch, as a very near approximation to the exact value of Δa in the case when no tensile forces are needed.

The exact expression for Δa, including the effect of the T's in shortening the axis is given in eq. (9), page 274 of Du Bois' *Graphical Statics*. It is seen from the following eqs. (12) and (13), giving values corresponding to our $\Sigma\Delta a$, how slight is the influence of this shortening of the axis.

This value of Δa having been assumed, the resulting equations and conditions for the arch fixed at the ends are the same as for the solid arch; so that the treatment pertaining to the case just examined applies.

36. Let us consider the segmental arch of 75 feet span, and 17 feet rise, given in art. 48 of *Theory of Voussoir Arches, &c.*

In Fig. 8, is shown a slightly exaggerated drawing of the arch with its load of 3,000 lbs. per foot, and the position of the pressure curve (as taken from a large drawing), corresponding to the maximum and minimum thrust in the limits of the middle third very nearly.

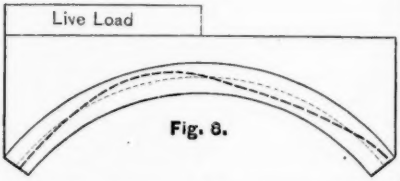


Fig. 8.

The above curve then corresponds to the trial curve p of Fig. 6.

If the greatest accuracy attainable by the graphical method is desired, the curves p and c (Fig. 6) should be drawn exactly as detailed in *Voussoir Arches*, whether we are treating solid or voussoir arches, uniform or isolated loading.

This remark applies also to many cases yet to be examined.

Now the loading being continuous in Fig. 8, the pressure line is a true curve so that after finding the centers of pressure on the joints, a curve can be traced through them, by the aid of some paper curves described with about the radius of the arch.

We thus have a curve, which is almost exactly the equilibrium polygon p due to the continuous loading.

With this curve p, we proceed as before to find the true curve c. The arch ring was divided into 16 equal parts, and ordinates drawn through the center of each portion as in Fig. 6.

It is found that the true curve c, differs so little from the curve first drawn, that it may be taken for it, in investigating the stability against rotation of this arch. Thus the statement made in the former treatise, that "it seems highly probable, that the *actual* line of pressures is confined within such limiting curves, approximately equi-distant from the center line of the arch ring, that only one curve of pressures can be drawn therein, corresponding, therefore, to the maximum and minimum of the thrust in the limits taken," is verified in this instance.

The similarity in the curves so found, might have been anticipated from a consideration of the conditions $\Sigma M=o$, &c., especially the first.

The above principle will be found of utility in constructing trial curves of pressure. If, in any trial curve, the two conditions, $\Sigma M=o$, $\Sigma Mx=o$, happen to be realized, we see from the previous construction (Fig. 6), that the third condition $\Sigma My=o$ only involves a change in the horizontal thrust, together with a change in the length of ordinates, from the line k_1k_2 to the curve c which is readily effected.

37. The theory pertaining to solid arches is only applicable to voussoir arches, *when the actual curve of pressures lies in the inner third of the arch ring—the face of the voussoir being rectangular—with no mortar joints, the stones, moreover, fitting perfectly before the centers are struck.* It is approximately true for thin mortar joints ($\frac{1}{16}$ inch, say) that are allowed to harden before the centers are struck, the mortar being of the best quality.

The above seems to be the most exact solution of the stone arch for the cases assumed that has yet been proposed; though it seems scarcely necessary to enter into it in testing the stability against *rotation* of such an arch, since this cannot happen theoretically, unless the arch ring is so small that one curve of pressure can be drawn in it; so that if only one curve of pressures, even, can be inscribed, in the middle third say, although it may not be exactly the true one, yet it would indicate that the arch was stable, since it cannot fall until all of its cases of stability are exhausted. If, however, we desire to know whether the limit of elasticity has been passed in any voussoir, then the construction for solid arches, if applicable, had best be made; and the maximum strain sustained by any "fiber" be determined by the use of eq. (4), art. 11.

38. It is evident that *concrete* arches, fall under the same treatment as solid arches of iron or other material. For such arches the question of *strength* is the only one necessary to consider, unless the *stability* of a pier or abutment is in question.

The spandrels of concrete arches, although generally built open, are nevertheless constructed as a part of the arch proper, the whole constituting one mon-

olithic structure. The effect is to modify the previous construction somewhat, though possibly on the side of safety. The same remark applies to any spandrel bracing in iron, etc., arches.

LOADING—VARIATIONS IN EI, ETC.

39. Let us again refer to Fig. 6, and consider the character of the loading, as well as the design of the arch, with reference to the most accurate construction of the curve of pressures.

If the loads bear at the isolated points a_1, a_2, . . . , the moments at one of those points equals, $H\overline{xac}$, $\overline{ac}$ being the vertical distance between the two curves a and c at this point.

But the object is, to ascertain approximately at each point, a_1, a_2, . . , the average moment, M on the part of the arch s, of which that point is the middle.

The above construction is thus only approximately true, since the ordinates at the points a_1, a_2, . . are generally greater than correspond to the average M mentioned.

Now it is by no means necessary to suppose the loads as acting at the points a_1, a_2, . .; in fact, it is generally most convenient, to suppose the loads as acting at the same horizontal distance apart, as actually happens in most iron arches with open spandrels.

Even with continuous loading, the ordinates ac do not give a good average for the part s, when the loads are supposed to bear at a_1,a_2, . . , or at uniform distances apart measured along the arc, the true pressure curve passing slightly below the apices c_1,c_2, . . The above remarks of course apply equally and primarily to polygon p.

Therefore, it is generally best, to draw polygon p (or curve p, if the loading is continuous) by supposing the loading applied at other points than a_1,a_2, . . ; when the parts of the ordinates, drawn through a_1,a_2, . . . , intercepted between vv' and curve p, will give the lines proportional to M_c more correctly than before. The residual small error can only be diminished by increasing the number of divisions of the arc, which remark applies in all cases.

40. If in place of dividing the arc into equal parts, we divide the *span* into equal parts, s in eqs. (7), (8), (9), &c., is no longer constant; so that if E and I are constant, the conditions for an arch fixed at the ends would become,

$$\Sigma Ms=o,\ \Sigma Mxs=o,\ \Sigma Mys=o.$$

Similarly for the supposed girder and arch acting as an equilibrium polygon.

If we divide the above equations by the horizontal distance between the loads, we see that each M must be increased in the ratio of the secant of the inclination at the point, in these, and the auxiliary equations, similar in form to the above. Thus in fig. 6, the ordinates of polygon p, as well as of curve a, must be increased in this ratio, after which the preceding methods apply in finding the position of mm' and k_1k_2, as well as the horizontal thrust.

This method introduces one advantage, with the several disadvantages: the consideration of moment areas in place of the single ordinates, since these ordinates are now proportional to the areas included between them and the equilibrium polygon.

41. If the quantities E and I of eqs. (7), (8) and (9) are variable also, we may divide the *span* into equal parts, and after drawing polygon p, alter the ordinates from the closing lines of p and a in the ratios, $s \div EI$, or in ratios proportional to these variable ones.

42. Now it frequently happens that the arch is increased in size towards the abutments, on account of the increased strain as we near the springings. If the ratio $s \div EI$ is constant for each length of the arch, having the same horizontal projection, then it is not necessary to alter the lengths of the ordinates of curves p and a, and the previous construction holds, when the span and not the arch is divided into equal lengths. This hypothesis ($s \div EI$ constant) was adopted by Prof. Greene in his analytic treatment of the parabolic arch in *Engineering News* for 1877, and is the basis of all graphical constructions founded on a division of the span into equal parts when the ordinates p and a are *not* elongated, as note Prof. Eddy's treatment in "New Constructions in Graphical Statics." For very flat arches, even when E and I are constant, this method is sufficiently correct. It will be illustrated further on.

43. When E and I are variable, in place of the construction of art. 37, we may divide the *arc* into parts of such

lengths, that $\Sigma s \div EI$ is the same for each length.

The division of the neutral line of the arch may be made by trial. Thus lay off a length s from the crown; change this (generally an increase) for the next length in the ratio of EI for the two portions; similarly for other portions. If the last division does not exactly reach the springing, divide the interval by the previous number of divisions. On increasing the length of the first division at the crown by this amount, and repeating the first operation, since each length is increased now in the same ratio (EI being the same for the same division nearly), the total increase is equal to the interval above, so that the last division should now just reach the springing. In arches, the sizes of the pieces do not vary so much, that the variation of E need be regarded, however different this may be in continuous bridges (see Bender's "Continuous Bridges"). It was included above, so as to present the subject in its most general form. We thus see how remarkably general this graphical method is; enabling us, with the same ease, to treat any form of arch, with varying section or otherwise, loaded in any conceivable manner.

44. The positions of the live load, causing maximum strains in the flanges and web of an arch, can be found by treating separately each weight and then combining, for any piece, the maximum strains that can be sustained by it for any disposition of the load. For a single weight the pressure curve or polygon becomes two straight lines as in Fig. 5, so that polygon p in Fig. 6 is quickly drawn, when the construction proceeds as before.

It will be observed that for the same arch the positions of T and T′ are the same for each weight, so that it is unnecessary to draw the polygon 1′2′..8′ but once. Similarly in Fig. 7, the construction for the curve a, holds for all the weights, as also the position of the closing line $k_1 k_2$ (Fig. 6); so that the constructions are thereby simplified.

FLAT ARCHES.

45. When the arch has a small rise compared with the span, it is evident that on dividing the *span* into equal parts, drawing ordinates, &c., that these ordinates will almost coincide with those found by dividing the arc into equal parts as previously done. When this is the case, the following neat construction applies:

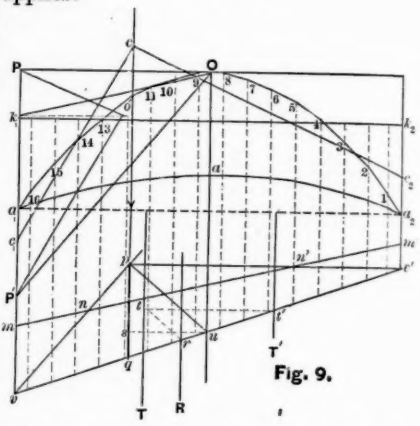


Fig. 9.

Let $a_1\ a\ a_2$ (Fig. 9) be the neutral line of a circular arch rib of 68.′404 span, 6.′03 rise and 100.′ radius, the half central angle being therefore 20°. Divide the span into 16 equal parts, and at the middle of every division erect ordinates as shown by the dotted lines.

The arch is so flat that it is best to increase the length of the ordinates, say four times, so that the neutral line takes the position 1, 2, ... This simply involves a corresponding decrease of $\frac{1}{4}$ in the pole distance in curves a and c according to art. 4, as we shall see more clearly as we proceed.

Let a single weight act at c, represented in intensity by the vertical PP′. Assume some point as o for a pole; then from some point v in the vertical through a_1 draw $vp \parallel oP'$ to intersection p with the vertical through c, then $pv' \parallel oP$ to intersection v' with the vertical through a_2; also draw the straight line vv'. We have first to find the true closing line mm' from the two conditions $\Sigma M_c = o$, $\Sigma M_c\, x = o$. Now since the span has been divided into equal parts, the ordinates from mm' to polygon vpv', that are proportional to M_c, are also proportional to the *areas* of the trapezoids of which they may be considered the medial lines, the horizontal altitudes being supposed the same. Now if we suppose the number

of ordinates indefinitely increased, ΣM_c above and below mm' approach the actual areas of the triangular spaces included between mm' and vpv' as near as we please. Hence the most accurate result attainable by the graphical method, in this case, is when we employ moment areas in place of the corresponding ordinates. The same remark applies to curve a regarded as an equilibrium polygon.

46. Reasoning as before, the condition $\Sigma M_c = o$, indicates that the sum of the areas vmn and $v'm'n'$ must equal the area npn'. Adding to both sides of this equality, area $vnn'v'$, we deduce, area $vmm'v'$ = area vpv'. Conceive a line drawn from m to v' dividing $vmm'v'$ into two triangular areas. Their centers of gravity are on the verticals T and T', drawn $\frac{1}{3}$ span from a_1 and a_2 respectively. Draw a straight line from p to the middle u, of vv'; the center of gravity of the triangle vpv' is at $\frac{1}{3}pu$ from u; the vertical R passing through it. The condition $\Sigma M_c = o$ is satisfied when the sum of the two triangles composing $vmm'v'$ is equal to the triangle vpv', and $\Sigma M_c x = o$ when the common center of gravity of the two triangles coincides with R, the reasoning being similar to that of art. 28.

These triangles having the same altitude, the span, are proportional to their bases vm, pq and $v'm'$. Therefore let pq represent the large triangle; from some point u as a pole draw uq and up; then, say from the intersection of qu with R at r draw rt and rt', parallel to up and uq respectively, to intersections t and t' with T and T'.

A line $us \parallel tt'$, divides pq into two parts, ps and sq, that are equal to the true values mv and $m'v'$. This is evident, because if ps and sq are forces acting along T and T', then by the construction of the equilibrium polygon trt', we see that $R = pq$, acting through r is their resultant. Thus the first two conditions are satisfied. We may now draw a line through o parallel to mm', to intersection with PP', which thus divides it into the two vertical components of the reactions.

47. We proceed now for the exaggerated curve of the arch 1, 2, ... exactly as before, and thus locate the position of the closing line k_1k_2.

In finding $\Sigma M_a y$ and $\Sigma M_c y$, as in art. 32, it may prove a convenience, to lay off the ordinates, proportional to M_a, or to M_c, along the line oP already drawn; then on drawing horizontals through 1 and 16, 2 and 15, etc., and choosing a pole as a, we proceed as in art. 32 to find $\Sigma M_a y$. This product having been found and laid off, the construction pertaining to it alone may be erased. On the line oP, or a parallel line, and the *same pole distance*, find the product $\Sigma M_c y$ as before. The moments M_c are proportional to the ordinates intercepted between mm' and vpv', as in the previous case.

It is of course well to ink all the lines that are to be used again, for another position of the load, which can appropriately be done in red or blue ink, to contrast better with the grey pencil lines.

48. From the construction last mentioned, we find that the ordinates of the equilibrium polygon p have to be elongated in the ratio of k_1c_1 to vm; and the pole distance is decreased in the inverse ratio. Since the closing line k_1k_2 is horizontal, we draw a horizontal through the point of PP' that separates the vertical components of the reactions to o', the new pole distance.

On elongating the ordinates mv, $m'v'$, the proper amounts, and laying them off from k_1 and k_2 to c_1 and c_2 respectively, we find the pressure curve $c_1\ c\ c_2$, by drawing $c_1c \parallel o'$P' and $c\ c_2$, which last should be parallel to o'P. The point c may also be found by elongating the ordinate at p to mm' and laying it off above k_1k_2 to c. On drawing lines from P and P' parallel to c_1c and cc_2, we find the pole o', which should agree with the first determination; though very slight errors of this kind may be neglected.

The ordinates may be changed by proportional dividers if at hand, or in the usual geometrical manner. When the lines are inconveniently short or long, take multiples or fractions of them. If one of the ratio lines is changed in the same proportion as those laid off along it, the result will be the same, in finding the true distance on the other ratio line.

49. The previous construction gives the reactions o'P' and o'P acting at their points of application c_1 and c_2, which is all the data required to ascertain the strains in the arch due to the load at c, as we shall see further on. The actual reactions for the real arch

$a_1 a a_2$ correspond to a horizontal thrust four times as great as the above, the distances $\overline{a_1 c_1}$, $\overline{a_2 c_2}$ being diminished one-fourth, and are given below for various positions of the load, as found from a large scale drawing of 4feet to the inch.

Let a denote the half central angle, on the angle between the radii at a_1 and a, or a_2 and a. Let β denote the angle from the vertical radius through a to the load at the neutral axis, measured at the center of the arc.

In the above figure we have taken the isolated weight PP′ equal to 10 tons, $a=20°$, and radius=100 feet, span 68.4 and rise 6. feet of the center line of rib.

We find by careful construction on the large scale drawing, the following values, for the vertical component V of the reaction farthest from the load, the horizontal thrust H and the distances a_1c_1 and a_2c_2, these distances being plus when laid off above a_1 or a_2.

β	V	H	$\overline{a_1 c_1}$	a_2c_2
	Tons.	Tons.	Feet.	Feet.
0°.	..5. ...	..27.08..	+0.87..	+0.87
	(5).	(26.69)	(0.82)	(0.82)
4°.	..3.53..	..24.60..	+0.1 ..	+1.39..
	(3.5)	(24.56)	(0.2)	(1.36)
8°.	..2.16..	..18.8 ..	−1.25..	+1.8
	(2.13)	(18.7)	(1.32)	(1.74)
12°.	..1.1 ..	..11.8 ..	−3.40..	+2.15
	(1.0)	(10.8)	(3.96)	(2.02)
16°.	..0.29 .	.. 3.4 ..	−11.0 ..	+1.82
	(0.27)	(3.36..	(12.0)	(2.25)

The numbers in parenthesis give the corresponding values obtained from Winkler's table mentioned below. We thus perceive in this case what reliance may be placed in a graphical solution, which was quickly made and not revised.

There were only 16 ordinates drawn, as in fig. 9; but with a greater number the result may be made as accurate as we wish, although the labor of construction is increased very much by using a large number of ordinates. Still there seems no other way of reducing error, especially for the loads near the abutments. The use of moment areas, as shown in fig. 9, involves but little approximation in principle for flat arches but in establishing the closing line k_1k_2 and the equality $\Sigma M_a y = \Sigma M_c$, we are forced to use the ordinates in place of moment areas, which process involves an approximation which is nearer the truth the greater the number of ordinates.

50. Having found the reactions at the abutments for a number of loads, the resultants of these reactions must be in equilibrium with all the loads; so that if we find their position at the abutments, on combining one of these resultants with the loads, the final resultant should be equal and directly opposed to the resultant found at the other abutment. Moreover, on the principle that the moment of the resultant is equal to the sum of the moments of its components, *the moment at any point of the arc is the same whether we find the moments due to each single load, as above, and combine them, or whether we combine the reactions for the several loads into one, at each abutment, draw the resulting pressure curve and find the resulting moment at the point of the arch considered.*

To find the point of application of this resultant reaction at one abutment, resolve each reaction due to a single weight into vertical and horizontal components. The former pass through the end of the arch, and produce no moments about it. Hence taking moments about this point, we find the point of application of the resultant reaction below or above the springing, equal to the sum of the moments of each horizontal thrust divided by the sum of the horizontal thrust. If now we draw the pressure curve from the point thus found, the horizontal thrust being equal to the sum of the thrusts due to each single weight, *this new pressure will satisfy the conditions of an arch fixed at the ends:* for the separate pressure lines for each weight satisfies the conditions $\Sigma m=o$, $\Sigma mx=o$, $\Sigma my=o$, $\Sigma m'=o$ &c., m,m', &c., being the moments at the points $a_1,a_2,\ldots$; hence the resultant pressure line will satisfy the conditions $\Sigma M=o$ &c., where M at any point is equal to the sum of the m's at that point (as just shown), as we find by simple addition of the eqs. for the separate weights. It is on this principle that we are enabled to consider separately the stresses due to loads, temperature etc., etc.

51. We see from the foregoing table that the horizontal thrust diminishes as the load is nearer the abutment.

It is well to assume such poles, that the ordinates M_c are of a convenient length; the pole being taken nearer the abutment as the load approaches it.

52. For the *exact* analytical treatment of the *circular* arch, the reader is referred to DuBois' *Graphical Statics*, pp. 271 to 311. The exact formulae for the distances $\overline{a_1c_1}$ and $\overline{a_2c_2}$ are given on p. 297, being due to Winkler.

On substituting the values of M_o, H and V taken from the tables, the term $k = \frac{I}{Ar^2}$ (where I is the Moment of Inertia of the constant cross section, A its area, and r the radius of the circle) being disregarded, we have the following table for the quantities $\overline{a_1c_1}$ and $\overline{a_2c_2}$, distances measured above the springs being plus. In the table, α is the half central angle, and β_1 the angle from the crown to the load given in terms of α; h is the rise, all referring to the neutral line of the rib:

Values of $c_1 = \overline{a_1c_1}$

β	$\alpha=10°$	$\alpha=20°$	$\alpha=30°$	$\alpha=40°$
0	+ .1343	+ .1370	+ .1417	+ .1488
.2	+ .0008	+ .0034	+ .0072	+ .0142
.4	− .2217	− .2196	− .2167	− .2106
.6	− .6667	− .6655	− .6646	− .6621
.8	−2.0020	−2.0015	−2.0148	−2.0142
α	h			

Values of $c_2 = a_2c_2$

β	$\alpha=10°$	$\alpha=20°$	$\alpha=30°$	$\alpha=40°$
	+	+	+	+
0	.1343	.1370	.1417	.1488
.2	.2232	.2262	.2307	.2381
.4	.2869	.2896	.2943	.3019
.6	.3345	.3371	.3422	.3493
.8	.3716	.3748	.3795	.3826
α	h			

Each of the members in the table must be multiplied by h, the rise of the center line.

The "approximate formulæ" given (p. 261) for $\overline{a_1c_1}$ and $\overline{a_2c_2}$, do not agree with these nearly exact values (k being very small) and are probably erroneous,* as might have been anticipated; since the term involving k is of undue importance. With very extensive tables (which would of course involve great labor in preparation) the solution of the solid or braced arch becomes a very simple matter. By interpolation, the preceding tables may be made to do good service. In the chapters just quoted is also the equation of the locus of the point c (Fig. 9). With the points c_1, c and c_2 thus found, the reactions are determined completely, so that the complete solution of the solid or braced arch is thus effected with great ease and rapidity by this combination of graphical and analytical methods.

53. Having found the reactions and their point of application for each single weight acting on the arch, we find, as in art. 11, or art. 82, further on, the strains in every division s, both of flanges and web and tabulate them. From such a table, we readily find the position of the live load that causes maximum strains in any part of the arch, and can thus tabulate them. For illustrations of this method of treatment see DuBois' *Graphical Statics*, p. 374 and on.

For the voussoir arch this determination of maximum stresses is not of so much importance as finding those positions of the loads that cause a maximum departure of the curve of pressures from the center line at various points. Prof. Greene, in *Engineering News* for 1877, p. 178, has given a table of actual moments at 19 points of a solid parabolic arch, due to a single load, placed successively at each point of division. As he states, "the greatest possible positive M, as well as the greatest possible negative M, for any combination of weights, occurs at each abutment; positive maximnm when the span is loaded from the other abutment to and beyond the center one or two points" (*i.e.*, $\frac{1}{20}$ to $\frac{2}{20}$ of the span); "negative when the other portion of the span is covered." This position of the load for the circular arch may be inferred from the preceding tables for c_1 and c_2.

In a parabolic voussoir arch, in which no joints open &c., we must therefore assume the uniform live load as extending from one abutment about six-tenths, or slightly less, of the span, as an approximation; though it is possible that other positions of the live load corresponding to a less hoizontal thrust, may cause the pressure curve to depart even farther

* I have verified the *exact* formulae given in the first four chapters of Supplement to Chap. XIV of DuBois' *Graphical Statics* [save that B_2 on page 284 should equal $(4\alpha\cos^2\alpha)$ in place of $(2\alpha\cos^2\alpha)$], but not the tables; so that I cannot vouch for the last two tables given though I believe the tables on which they are founded to be correct.

from the center line, than the position just mentioned. This can only be determined by trial.

The same remarks apply to flat circular arches: in fact it may be well to test any style of arch by first supposing the live load to cover about half the spam from the abutment.

TEMPERATURE STRAINS.

54. It is usual to class under this head, strains due to a change of temperature above or below the temperature at which the arch is finished, (supposing all parts then to fit accurately); as well as strains due to the components T(art.9) tangential to the rib that compresses it, and cause it to be suited to a less span than it is fitted to; together with strains due to an actual lengthening of the span due to the compression.

Denote the temperature at which there is no strain in the arch due to temperature, as the *mean* temperature; and the greatest deviations from this, above or below, by $+t$ or $-t$.

Denote the expansion of the metal for a unit of length and one degree by ε. The total change of span is then $l\varepsilon t$, denoting the span by l.

This tendency to a change of span being resisted by the abutments, causes a horizontal thrust or tension there. Since for the *arch fixed at the ends*, in direction, &c., there will also be strains in the flanges at the abutments due to this change, there must also be a couple at the abutments to produce this effect.

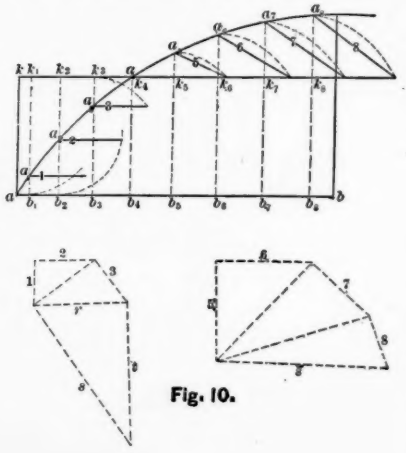


Fig. 10.

Thus in Fig. 10, representing half of the arch, suppose a horizontal force H acts at k; conceive two horizontal and opposed forces, each equal to H to act at a. This does not disturb equilibrium, but effects a transfer of H to a (where it must be sustained) and adds the couple $\mathrm{H}.\overline{ak}$. If the arch is symmetrical about the center, the same condition of affairs must exist at the other abutment, so that the force at k must act along the horizontal kk_s, as was assumed.

This follows again from the condition used to fix the line kk_s, which is, that the total change of direction of tangents, in going from one abutment to the other, is zero; *i.e.*, if E, I and s are constant, we must have

$$\Sigma \mathrm{M} = o,$$

which gives the same position to the line kk_s as found in art. 31.

Therefore having divided the *half arc* into any number of equal parts (8 in the Fig.) and drawing ordinates a_1b_1, a_2b_2, . . through the centers of each part, we have the moment at any point, as $a_3 = \mathrm{H}.\overline{a_1k_1}$, etc; the moments changing sign on opposite sides of kk_s.

55. The object now is to find H.

The effect of the change of temperature is to produce a virtual alteration of span (this being small), so that (eq. 8) of art. 13 must be satisfied. We have then,

$$hl\varepsilon t = 2\frac{s}{\mathrm{EI}}\,\Sigma \mathrm{M}y;$$

the summation $\Sigma \mathrm{M}y$ being taken from the center to one end, it being the same for the other half of the arch.

The moments M, at the successive points a_1, a_2, . . , we have just seen, are equal to products of the type $(\mathrm{H}.\overline{ak})$, so that the preceding eq. becomes

$$\mathrm{H}\Sigma(ak.y) = \frac{h\mathrm{EI}}{2s}$$

If the known quantities h, I, s, $\overline{ak}$ and y are given in feet (say) in true or real dimensions, and E in tons (say), then on forming the above sum, and solving the eq., we find the value of H in tons. If E is given in pounds, then H is likewise.

56. The sum of the products $\Sigma\overline{ak}.y$, may be found by calculation or graphically as previously shown. The following familiar graphical construction (see

Chauvenet's *Geometry* pp. 120, 137) is a good one when the segments are not too dissimilar in length.

1/. For the part below kk_s, describe circles on k_1b_1, k_2b_2, ... as diameters; the intersections of the horizontals through a_1, a_2, .. with these, give the lines 1, 2, .., which measured to the scale of the arch and squared, give the products $\overline{a_1k_1}.\overline{b_1a_1}$, $\overline{a_2k_2}.\overline{b_2a_2}$, etc. Therefore in the Fig. just below this part of the arch, draw 2 perpendicular to 1, of the lengths above; the square of the hypotenuse of the right triangle is equal to 1^2+2^2. Next draw 3 perpendicular to this hypotenuse, we have then $r^2=1^2+2^2+3^2$.

2/. Next consider the part of the arch above kk_s. On $\overline{b_2a_2}$, b_3a_3, . . ., as diameters describe circles; from their intersections with kk_s, draw lines to a_2, a_3, . . .; and denote the lengths of these lines by 5, 6...; then as before find s^2=the sum of their squares.

3/. Draw a perpendicular t to r of such length that s is the hypotenuse of the right triangle whose three sides are r, s and t. Then $t^2=s^2-r^2=\Sigma\overline{ak}.y$. Hence measure t to the scale of the arch and square the number found, which square is thus equal to $\Sigma\overline{ak}.y$ required, whence H may be found from the preceding equation.

Example.—Let the span of the arch be 518 ft., the rise 51.8 ft., EI=39,680,000 foot tons, and let=0.2735 feet (ϵ is taken as .000012 for 1°c), t being 44°c (about 80° Fahrenheit).

On dividing the semi-arch, drawn to a scale of 20 feet to the inch, into sixteen equal parts, and proceeding as before, we find $t^2=\overline{62.8}^2=3944.=\Sigma\overline{ak}.y$, whence H=82.4 tons.

The analytical formula (see Du Bois' *Graphical Statics*, p. 302, eq. (66), omitting the term k, which is extremely small (less than $\frac{1}{75\,000}$ for the middle span of the St. Louis Bridge) gives H=81.9 tons, on substituting in it, a=.3947353, sin a=.3845639, cos a=.9230984 and r=673.4; a in degrees is put at 22°37′.

57. When E, I and s are variable, we may pursue the treatment of art. 41; *i.e.*, alter the distances k_1a_1, etc., for a trial line kk_s in the ratios $s\div$EI. After the position of kk_s is obtained we proceed as before, using the altered ordinates.

In fact, the preceding equation must now be written,

$$h=2H\Sigma\frac{\overline{ka}.y.s}{EI};$$

$\overline{ka}$ being the ordinates to the arch from the closing line. Now we see from this equation, that at each point a_1, a_2, . . ., that $\overline{ka}$ being multiplied by $\frac{s}{EI}$, changes the ordinates in the ratio before stated.

Now if EI is greatest, nearest the abutments, the changed ordinates ka become less there proportionally; so that the line kk_s is raised above its normal position, whence the Σ above may be less than before and H greater. This was the case at the St. Louis bridge, the true horizontal thrust due to temperature being 104 tons, the varying cross section being considered.*

58. We have seen that the strains due to a change of span, real or virtual, result from a horizontal force H at a, and a couple H.$\overline{ak}$ also acting at a. A rise of temperature, producing a virtual shortening of the span, the force H at a acts towards b. On the contrary, for a fall of temperature below the mean, or the elastic shortening of the arch due to the tangential forces, or for an actual lengthening of the span the force H at a acts towards the left. The couple H.$\overline{ak}$ is in the first case right handed; in the second case left handed. We see easily that the moment, at any point

$$a_2=H.\overline{ak}-H.\overline{a_2b_2}=H.\overline{k_2a_2},$$

as was asserted. In fact we must conceive two opposed horizontal forces each equal to H, at a_2, one of which, with the force H at a forms the couple, whose moment is H,$\overline{a_2b_2}$ as above. This gives us finally the moment H,$\overline{a_2k_2}$ at a_2, together with a single horizontal force at H, acting to the right or left according as H at a acts to the right or left. This force can be resolved into tangential and normal components at a_1,a_2, etc.; from whence the final stresses can be found as usual. If more convenient a single force H, acting along kk_s, may be used, being the resultant in position of all outward forces acting on the arch, and the strains in the flanges determined by taking moments about suitable apices.

* The entire construction above for finding H differs essentially from that given by Prof. Eddy in *Researches in Graphical Statics*, which is believed to be erroneous, as also the determination of temperature strains for the other arches treated by him.

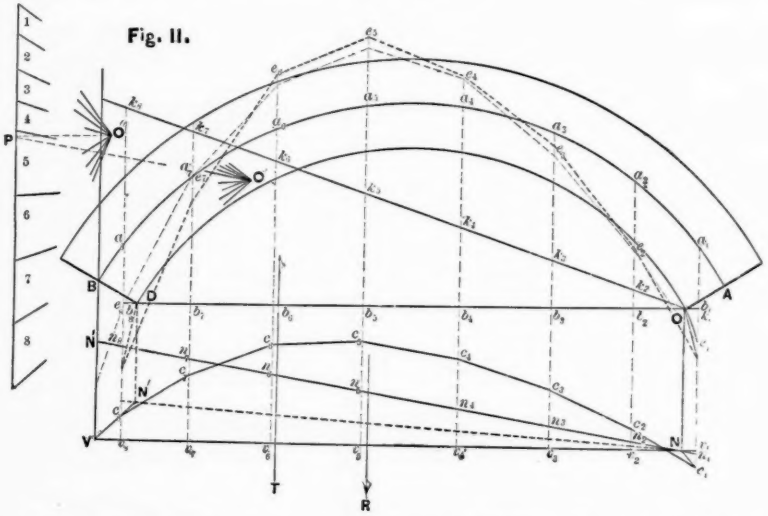
59. If preferred, the force H due to temperature, compression, &c., may be combined with the reactions due to any system of loads, according to the principles of art. 50; the resulting pressure curve, with a pole distance equal to that due to the loads, plus H, is then the true one by which to determine the strains in any member of the arch. In this way is included the effect of the components T (art. 9), hitherto neglected. The sum of the horizontal projections of the shortening of each part s of the arc, due to T, which is easily computed is the virtual change of span.

If the strain f, per square unit, due to T alone, is supposed approximately the same on each cross-section, the change of span is $\left(\frac{f}{E} L\right)$, where E is the modulus of elasticity and L the span, from which the resulting strains are readily found. This is generally a sufficient approximation.

The strains due to temperature, change of span, &c., are very large, requiring the most accurate fitting of the arch members. By diminishing the proportionate depth of arch; or using a material, as steel, with a high modulus of elasticity, these strains may be diminished.

60. We have treated the arch "fixed at the ends" with some detail, since its solution includes, to a great extent, that of arches with hinges. The latter will now be considered, as briefly as is consistent with clearness, avoiding, as much as possible, repetitions of the same construction.

Fig. II.



ARCH FIXED AT ONE END AND HINGED AT THE OTHER.

61. Let Fig. 11 represent an arch fixed in direction at B and jointed, or free to turn, about the point O at the other abutment. We have taken the point O outside of the neutral line *aa*, as this presents the most general case. In fact, when for an arch *with flat joints* at both ends, for any loading, the pressure curve passes outside of the arch at one abutment, when treated as an arch fixed at the ends, the arch there will rotate about the edge of the joint, as O, *nearest the pressure curve*; and must be treated as an arch fixed at B and jointed as O. If the pressure curve thus found, however, passes outside the joint at B, then the arch must be regarded as hinged at both ends for the load in question. If the pressure curve for the arch fixed at the ends passes outside of both joints B and A. We next try the pressure curve for the arch fixed at one end (the end where the pressure curve is nearest the neutral line), and if the resulting curve again leaves the arch ring at the end supposed fixed, then the arch falls under the head

of the "arch hinged at both ends," the points free to turn, lying on the abutment joints nearest the pressure curve. (See Mr. Pfeifer's article in VAN NOSTRAND'S MAGAZINE for June, 1876, p. 494.) If the arch is firmly bolted to the abutments at the ends, so that the connection can exert sufficient tensile resistances, as in the St. Louis bridge, there can be no *rocking* at either abutment, so that the arch is properly treated as fixed at the ends in all cases. It is evident how convenient the graphical method is in determining the special case to be treated even when the analytical method is finally applied.

62. The arch, Fig. 11, is taken of the same proportions and divided up in the same manner as Fig. 6. The true pressure curve e is shown by the dotted line, the force polygon being O′18 on the left. We shall presently show how to draw this curve. Granting that it is the true one, it possesses some peculiarities, well to note, about the point o, through which the right reaction of all the loads must pass in the direction e_1O. On combining this reaction (ray 1 P on the left), with the load at a_1, we get the resultant (ray 12), acting along e_1 e_2, on the part of the arch a_1a_2 and so on. Hence e_1e_2 does not pass through O.

We draw the equilibrium polygon c, as in Fig. 6, with an assumed pole o', except that at c_1, the direction of the reaction, or ray 1 P, is produced to intersection N with the vertical through O.

If we draw NN′ as a trial closing line, and suppose forces F=o'P (o'P being parallel to NN′) applied at N, N′, c_1 and V, as in Fig. 5 parallel to NN′, we see that ccNN′ is the equilibrium polygon due to the weights, with moments at B and a_1 proportional to VN′ and n_1c_1. It is seen that at each apex, N, c_1, c_2, .. V, N′, we have forces in equilibrium.

On conceiving the polygon cut about N, &c., as in art. 4, we see that the moment at N is zero, as we desire it to be.

The moment at B, necessary to fix the end of the arch, acting as a girder, is equal to H.$\overline{\text{N′V}}$, H being the horizontal thrust corresponding to a pole o', and $\overline{\text{N′V}}$ having its proper length, to be determined presently.

Now the true pressure curve of the arch must satisfy the conditions of art. 15,

$$\Sigma_B^A \text{M}x=o, \quad \Sigma_B^A \text{M}y=o;$$

the origin of co-ordinates being at the point O, x horizontal, y vertical.

63. We must likewise have for the "girder," since the vertical deflection of o below the tangent at B must be zero, $\Sigma \text{M}_c x=o$; whence $\Sigma \text{M}_a x=o$, as in art. 23. Now M_c is proportional to ordinates of the type $\overline{nc}=(\overline{vc}-\overline{vn})$; so that the preceding condition may be written, $\Sigma\,(\overline{vc}-\overline{vn})\,x=o \;\;\therefore\;\; \Sigma\,(\overline{vc}.x)=\Sigma\,(\overline{vn}\,.\,x)$. That is, calling the horizontal distances of $c_1, c_2, \ldots$ from $o, x_1, x_2, \ldots$, we must have, $(\overline{v_1c_1}.x_1+\overline{v_2c_2}.x_2+..)=(\overline{v_1n_1}.x_1+\overline{v_2n_2}.x_2+..)$ It is seen that although $\overline{v_1c_1}$ and $\overline{v_1n_1}$ are minus, yet x_1 is also minus (art. 12) so that all quantities in the above equation may be regarded as plus; so that the result is the same if $\overline{v_1c_1}$ and $\overline{v_1n_1}$ are supposed laid off a distance x_1 to the left of N, in the above summation. We can form the above products graphically as shown in art. 32, the parallels being already laid off, and if the two members of the equation are not equal, we have simply to alter VN′ in the ratio of the right to the left members, which thus fixes the position of the line NN′.

Otherwise, by aid of two equilibrium polygons, using $\overline{vc}$ and $\overline{vn}$ as forces, ($\overline{v_1c_1}$ and $\overline{v_1n_1}$ being laid off to left of N as above) find their resultants T and R similar to T and R of Fig. 6; then if t and r are their horizontal distances to N, we must have, Tt=Rr. If these products are unequal, alter VN′ as before. This method is best in view of what follows.

64. For the curve aa regarded as an equilibrium polygon, some line kk, passing through O must be the closing line; the ordinates of the type $\overline{ka}=\overline{ba}-\overline{kb}$ are then proportional to M_a, so that as before we must have the following condition satisfied, the quantities all being plus,

$$(-\overline{b_1a_1}.x_1+\overline{b_2a_2}.x_2+\ldots) = (\overline{b_1k_1}.x_1+\overline{b_2k_2}.x_2+\ldots)$$

These products, except the first, are all plus, and may be formed as above. Since the effect in the summation is the same if we consider b_1k_1 laid off, x_1 to left of O, all quantities being plus, we can use the second method above more easily in this case; for T′=sum of ordinates of type bk, acts in the same verti-

cal as T, so that if the first construction has been made, we have only to find T′, when T′t will be known. The resultant, R′=Σ(ba), acts through the crown, since b_1a_1, b_2a_2, . . , are all positive; the lever arm of R′ about o, r', equals the distance from the center of the span to o.

We should have, if k_1k_8 is in its proper position, T′t=R′r'; otherwise alter b_8k_8 in the ratio of R′r' to T′t, to find the true position of the closing line k_1k_8.

65. From the condition that the space is invariable we have,

$$\Sigma My=\Sigma(M_c-M_a)y=o$$
$$\therefore\ \Sigma M_c y=\Sigma M_a y;$$

the summation being extended over the entire span.

Writing the proportional equation in full, we have, in this case, putting $y_1=b_1a_1$, $y_2=b_2a_2$, etc.,

$$(-\overline{n_1c_1}\cdot y_1+\overline{n_2c_2}\cdot y_2+\overline{n_3c_3}\cdot y_3+\overline{n_4c_4}\cdot y_4$$
$$+\overline{n_5c_5}\cdot y_5+n_6c_6\cdot y_6-n_7c_7\cdot y_7-\overline{n_8c_8}\cdot y_8)$$
$$=\overline{(k_1a_1}\cdot y_1+\overline{k_2a_2}\cdot y_2+\overline{k_3a_3}\cdot y_3+\overline{k_4a_4}\cdot y_4$$
$$+\overline{k_5a_5}\cdot y_5+\overline{k_6a_6}\cdot y_6-\overline{k_7a_7}\cdot y_7-\overline{k_8a_8}\cdot y_8).$$

By way of variety, let us scale off the above distances from the drawing, using a scale of 40 feet to the inch.

On forming the sum of the products indicated we have for the ratio,

$$\Sigma M_a y\div\Sigma M_c y=\tfrac{42}{15}$$

Therefore each of the ordinates nc of the polygon c must be increased in this ratio; after which they may be laid off from the closing line k_1k_8, and the true pressure curve drawn as shown by the dotted line ee.

66. In order to find the new position o'' of the pole, for the true force polygon o''18 on the left, we draw from the assumed pole o', a line o'P ∥ NN′, thus dividing the load line, 18 into the two vertical components of the reactions. Then draw the horizontal Po'' equal to the old pole distance diminished in the ratio of 15 to 42, giving o'', the new pole, from whence the pressure curve may be tested, or drawn anew having one starting point.

In altering the length of lines in a given ratio, use may be made of proportional dividers, for small drawings, especially; or the common geometrical method of using two ratio lines may be employed, as explained before.

As a final test of this pressure curve, we find by scaling the ordinates, multiplying, &c., that Σ Mx=o, and Σ My is very nearly zero also. The ordinates to dotted curve above aa are of opposite sign to those below in this summation.

67. On comparing the above pressure curve with that of Fig. 6, we see that it departs much farther from the neutral line aa, though the horizontal thrust is much less. The depth of arch ring, compared with its span, has been greatly exaggerated for usual cases in practice; so that the point O is generally much nearer A. When O is to the right of the vertical a_1b_1, the construction is, moreover, slightly simplified, as there are no negative ordinates, v_1c_1, v_1n_1, b_1k_1, or abscissas x_1, for this case. This is likewise true when O is taken above a.

For different loading or position of O, the form of the pressure curve may be materially altered.

For the determination of maximum stresses, we have to treat single weights as before, and combine their effects.

68. *Temperature Strains.*—A real or virtual alteration of span produces strains, similar to those caused by a force acting at O along k_1k_8; since such a force, whose horizontal component, call H, causes moments of the type H.$\overline{ak}$ at each point a; therefore the condition that the vertical deflection of O be zero, Σ My= HΣ($\overline{ak}$.y)=o, requires the same closing k_1k_8 previously found, the moment at O being zero. Calling h the change of span due to temperature, compression, &c., we have as in art. 55,

$$H\Sigma(\overline{ak}.y)=\frac{hEI}{s};$$

from whence H may be found as before, noting that $\overline{ak}$ is minus on one side of k_1k_8, and plus on the other side in the summation.

69. For very flat arches, we may divide the span into equal parts, whence the ordinates corresponding to M_c and M_a are proportional to the areas of the trapezoids of which they are the medial lines, so that moment areas may be employed; which is most accurately done in connection with Simpson's rule. The reader is referred to Eddy's "Researches in Graphical Statics," for the most extended application to arches of moment areas.

The reader will do well to apply the

principles of art. 34, in applying the two conditions, $\Sigma M x = o$, $\Sigma M y = o$: especially as a new form of equilibrium polygon is introduced. The results are of course the same in both cases. The method of that article may be applied to all the following cases.

ARCH HINGED AT BOTH ENDS.

70. Let the same arch, fig. 11, be regarded as hinged at O and D, the lower edges of the abutment joints. The reactions must now pass through O and D. Having drawn a trial curve cc, extend V_c to intersection N″ with the vertical through D; the line NN″ is the true closing line for curve c. Hence drawing through O′ a line parallel to it, we divide the load line 1.8 into the two vertical components of the reactions.

To elongate the ordinates of polygon c to their true lengths, we have the condition that the span is invariable,

$$\Sigma M_b y = \Sigma (M_c - M_a) y = o$$
$$\therefore \Sigma M_c y = \Sigma M_a y = \Sigma (y^2);$$

the line OD being the true closing line.

On measuring $y = ba$, at each point $a_1, a_2, \ldots$, and the ordinates proportional to M_c included between NN″ and polygon c, noting that the ordinates at c_1 and c_8 are negative, and forming the above products we find that,

$$\frac{\Sigma M_c y}{\Sigma M_a y} = \frac{31}{103};$$

hence we must shorten the pole distance in this ratio, and lengthen the ordinates in the inverse ratio, as shown previously. The lengthened ordinates from NN″ to curve c, are laid off from $b_1 b_8$ giving the true pressure curve, as shown by the dotted line passing to the right of the first curve at e_8 and e_7 and crossing it about e_6 and e_2. When the hinged, or free to turn, points are at A and B there are no longer negative moments at c_1 and c_8.

71. *Temperature strains.* The thrust or pull H due to temperature &c., can only act along OD, giving moments of the type $H.\overline{ba}$; whence $Hs\Sigma(\overline{ba}.y) - hEI$ from which equation H may be found on substituting the values of $s, \Sigma(\overline{ba}.y) = 103$, h and EI as hitherto shown.

CURIOSITIES OF WATER SUPPLY.

From "The Engineer."

THAT water engineering—limiting the expression to the supply of water to the inhabitants of cities and towns—has retrograded in modern times, may be plausibly argued, and, indeed, in some of its aspects, that is an undoubted truth. Economy in outlay formed no element in the projects for the supply of water to ancient Rome. They were designed with an eye to public utility only, and whether executed at the cost of the State or by the munificence of consuls or emperors, had no divided aim, as in the case of our Metropolitan water supplies, to individual or joint stock profit: hence the prime aim of the Roman water engineer was to procure and pour into the city, often through miles of magnificent aqueduct, such a copious flood of pure, cool, and limpid water, that every citizen, poor or rich, without impost or payment, should be free to use it or waste it at his pleasure. A few of these magnificent waterworks still bring their supplies into Rome, and the remains of like works are to be found still scattered over every part of Europe to which Roman civilization had reached. But with the decay of Roman power a sad falling off began, and in those ages of ignorance and superstition which so many amongst ourselves are willing to extol as the "ages of faith," water supply was everywhere neglected except in the monasteries or other great ecclesiastical centers, and the supply of water throughout the cities and hamlets of Europe dropped to the lowest point that natural conditions would permit. Whoever reads "Haecker's History of the Epidemics of the Middle Ages," translated some years ago from the original German by the Sydenham Society, will easily recognize how much their virulence and spread was due to the insufficiency

or badness of the water everywhere consumed. We may learn from Falstaff's experiences of the "buck basket," in the "Merry Wives of Windsor," how filthy habits were produced by the difficulties of ablution arising from water having to be carried into the houses, for greater or less distances, from the insufficient public source. In London this culminated in the "plague" and in the "great fire," and after an interval of apathy, came the epoch of the modern water supplies of London, the earliest being the London Bridge pumping engines driven by undershot water wheels, actuated by the Thames pent up at old London Bridge, and the magnificent work of Sir Hugh Middleton, still usually the bearer of the best water that comes into London; and then came the false political economy of conferring legislative powers upon collections of private individuals, miscalled public companies, to supply water and levy rates for it, the first necessary of life, which should make it a source of profit, a false system from which all Londoners are now suffering. Our water supply is tolerable in quality, and admits but of little complaint as to quantity, but it has, by the vicious system of the water companies, been made extravagantly dear, and as surely as but recently the false system of toll-bridges has been abolished, so will a not very distant time arrive when the London water companies will have to be bought up. The debate upon this subject, initiated by Mr. Fawcett on Wednesday, was very significant as to this. We do not propose, however, to pursue this large subject systematically, or at any length; our aim is simply briefly to place before our readers a few of the curiosities, as they present themselves to the engineer at least, of water supply to communities, small or great. Probably no community, at least in Europe, is more strangely supplied with water than are the inhabitants of Arenara in the volcanic island of Panaria, one of the Lipari group. From nearly the summit of the extinct volcano which forms the entire island descends an unbroken field of black volcanic sand, consisting chiefly of minute crystals of pyroxene, which covers everything beneath to a depth of several feet and descends to the seashore, where the waves have at two or three points exposed the beds of lava or other volcanic rocks on which the black sand reposes; the total surface thus covered probably not exceeding, if indeed it reaches a square mile in area. The entire water supply for several hundreds of the inhabitants of the island is dependent upon the water which drips continually at near the sea level from this one bed of volcanic sand. Hooped or earthen vessels are placed to receive it, and, though unguarded, are reverently preserved from pollution or injury by the many who come to fill their own vessels from this dripping well. The supply from it varies but little at any season, or in any weather; when the surface of the black sand is so heated by the sun as to be painful to the foot through a thick sole, and when the rain descends in the torrents characteristic of the climate of this part of the Mediterranean, the threads of water none bigger than a pencil, continue to flow tolerably cool, good in taste, and perfectly pellucid; they are never turbid; it is, in fact, a natural filter bed. In a small sea-surrounded volcanic island, where there is no running water, nor indeed any water except what is husbanded at Singua and elsewhere in cisterns, the precariousness of the supply obtrudes itself more than its smallness; yet the testimony of the priest and of other of the more intelligent inhabitants is, that within the present century it never completely failed, and but once became so diminished that boats were sent off to the much larger adjacent island of Lipari to bring back a supply. There are some curious superstitions as to their singular water supply amongst the Panariots, but they are not worth notice here. Venice, though upon a far larger, and applying to a greatly larger population, was, until a comparatively recent period, dependent upon something of much the same sort of precarious supply, though in this case the supply was obtained, not from unaided nature, but by human ingenuity. The subjacent soil of the Queen City of the Adriatic consists everywhere of beds of silt and sand, permeable by water, and thoroughly waterlogged. To obtain a supply of water from wells in such a soil was of course impossible. In comparatively recent times attempts have been made to sink artesian wells through the silt, and into the formations underlying the lagoons and bed of the Adriatic; but without success. Beds of sand and of

sandy peat, sloping steeply seaward, were encountered to a depth exceeding 500 ft.; but no potable water. The first introduction of extraneous water entered the city with the railway viaduct, about thirty years ago; and this source of supply has, we believe, been since considerably increased; but prior to that the supply of water to Venice was only what it had been from the fifteenth century, and earlier, in the days of her opulence and political importance. It consisted simply in the rain-water collected from the roofs and court yards of her palaces and houses, and from the surfaces, which were carefully flagged and channel-coursed, of her public places, such as the well-known Piazzo di San Marco. The water thus collected from clean but very limited surfaces was conducted by covered pipes or channels into ingeniously constructed draw-wells, which were at once filtering cisterns and reservoirs. They were thus constructed:—An excavation in the shape of an inverted truncated pyramid with a level bottom was sunk in the soil and lined up to near the surface by a timber frame and close boarding, which was caulked water-tight. From the center of the flat bottom was built up a cylindric well of dry brickwork, and the space between it and the timber lining was filled up with stones and gravel and sand sufficiently fine to form a filter bed. The rain-water from the surrounding roofs or flagged spaces was conducted into this space, overflow channels being provided to take off any surplus rain after the filtering space and well were filled. The whole was covered over and flagged upon hard wood beams, the mouth of the well was sometimes covered over also, but in the more important—those situated in public places—it was parapetted by a superb circular casting in bronze, often richly adorned by figures, in high relief. Several of these superbly-finished draw wells still exist in Venice and in other cities of the ancient Lombardo-Venetian territory. The water collected in these wells was potable and inoffensive to the taste, to an extent which could not have been expected from water thus obtained, and in such close contact with the stagnant and more or less putrid waters everywhere filling the soil beneath the city. It is to be remembered, however, that Venice was never a smoky city, and that a large proportion of its offal and solid sewage was taken away by the canals and applied as manure, or boated away, and thrown into the "lagoons" or the open sea. Still the presence of fevers and various malarious diseases, and malignant forms of organic disease attested, and still attest the unhealthiness of the position, and the want of an abundant supply of pure water and an efficient system of deep drainage. For all culinary and domestic purposes these curious wells were the only resource, but in the opulent days of the city a very perfect system of water-carriers was in operation; these brought water for all other purposes from the Grand Canal or the Brenta. Venice was a salient example of a city ill-posted from the outset; and gradually made great by many circumstances; but in Europe at large it is a rare thing to find any ancient town, or indeed any modern city, as well supplied with water as are some of the great cities of the United States, such as Chicago and New York. Nature here places in the hand of the engineer capabilities such as are denied him, for the most part, in the civilized regions of our hemisphere. In these new American cities, and in those of our Australian colonies, will be found hereafter the noblest examples of civic water supply, and to which the water engineer of the future will look for instruction and example even more than to the superb waterworks of the days of Roman power. Already the construction of the Geelong and Sandhurst water reservoirs, of which some account will be found in Vol. 56 of "Minutes of Proceedings of the Institution of Civil Engineers, 1878-79," has solved upon a great scale a problem of great and widely-varied interest, namely, whether blown sand absolutely free from clayey or vegetable mixture can be made to impound water, and it has been proved that at a certain thickness a bank of such sand, though its interstices are completely waterlogged, become absolutely impervious to the passage or percolation of water through it.

A complete theoretic account of the unanticipated facts remains yet to be given, although it can be easily discerned that—just as any tube of small bore may be so prologed that a given head of water may be unable to force its way through

it, so the capillary interstices between the particles of pure silicious sand, when filled to a certain distance with water, may have the small resistances gradually accumulated until they permit none to pass. As always happens, as a consequence of the discovery of a new truth, whether of observation or experiment, practical deductions of an important character have already followed from the construction of these sand-made reservoirs in Australia. The sand embankment surrounding such a reservoir becomes itself an immense magazine of water, so that when the surface is lowered by the afflux at any time exceeding the supply, the volume of water drawn off is found by calculation much to exceed the visible capacity of the reservoir which has been emptied. Where the hight of the embankment is not too great and the material, pure sand, exists, there will be a positive advantage in constructing reservoirs of sand only, in place of clay, or other impervious material.

THE HYPOTHESIS OF AN ABSOLUTE ZERO OF TEMPERATURE.

Translated from "Revue Industrielle" for VAN NOSTRAND'S MAGAZINE.

The temperature of the assumed absolute zero is, as we know, determined by the following reasoning;

H being the pressure of a gas at the temperature t, and H_0 the pressure at zero, these quantities sustain the relation expressed by the equation

$$H=H_0(1+\alpha t),$$

α being the coefficient of dilatation of the gas.

If we wish H to become nothing, it is necessary that $1+\alpha t=0$ (H_0 being a constant) so that $t=-\frac{1}{\alpha}=-273°$.

This reasoning is faulty in many ways. The coefficient α of dilatation is not the same for all gases; especially is this true near $-273°$ where the so-called permanent gases approach the point of liquefaction. Furthermore this coefficient varies with the temperature in a manner, at present, not well defined.

Consequently there exists for each gas a different absolute zero, whose value can not at present be determined.

But even if we admit that this coefficient is practically the same for all gases, and that it does not vary with the temperature, the above equation is still erroneous.

Consider for instance a gas whose volume is unity at a certain temperature. If the temperature be raised one degree, the volume will become $1+\alpha$: if raised one degree higher its volume becomes $(1+\alpha)+\alpha(1+\alpha)$ or $(1+\alpha)^2$. The volume for a temperature t becomes $(1+\alpha)^t$.

If this be developed by the binomial theorem we get the following expression;

$$V=1+\alpha t+\alpha^2\frac{t(t-1)}{1.2}+\alpha^3\frac{t(t-1)(t-2)}{1.2.3}+\&c.$$

The first two terms represent the usual formula; the other terms being neglected.

Now although we may, for practical purposes, reject the remaining terms, it is clearly improper to do so when we extend the application of the formula to extreme limits.

The equation $H=H_0(1+\alpha t)$ is incorrect.

The true one is; $H=H_0(1+\alpha)^t$

In this expression if we make $H=0$, we find $t=-$ infinity.

We may say, then, that in the present state of our knowledge, nothing justifies the assumption of an absolute zero, and that it is one of those false hypotheses that tend to retard the development of science.

ON THE FLOW OF WATER IN RIVERS.*

By DE VOLSON WOOD, C. E.

Transactions of American Society of Civil Engineers.

At the Convention of the Society in June of last year, held in Boston, that eminent hydraulician, James B. Francis, reported the result of an experiment made by him, which showed that the water at the bed of a stream of water gradually rises to the surface. He placed a white substance, having about the same specific gravity as water, at the bottom of the stream, leaving it in such a condition that the moving water would take with it some of the substance; and he observed that the water thus colored rose to the surface at a point some distance below where it was placed. This, so far as I know, was the first definite statement of this mode of action in streams of water. In the discussion which followed, I stated that admitting the correctness of the principle stated by Mr. Francis, it followed that the water at the surface must necessarily find its way to the bed of the stream, and that this action might enable us to account for the fact, now generally admitted, that the point of maximum velocity in the cross-section of a flowing stream is not at the surface, but at a lower point, generally between the surface and the half depth of the stream. My view, as then expressed, was that gravity acting upon the particles during their descent would accelerate, or at least would tend to accelerate, their motion; and that the action of gravity, combined with the motions impressed by the mutual actions of the particles, might account for the position of the point of the maximum velocity. This view I still believe to be correct, and I desire in the following paper to consider more in detail some of the principles herein involved.

The earliest experiments showed conclusively that the velocity at different points in the cross-section of a stream was not the same—that it was greater near the middle than at the sides of the stream, and greater at the surface than near the bottom. Improved apparatus and more careful experiments led to the statement that the maximum velocity was at the surface, and that it decreased with the depth, though not as a direct simple function of the depth. Some assumed that the curve of velocities would be represented very nearly by a parabola whose axis was vertical and whose vertex was at some unknown distance below the bed of the stream (see D'Aubisson's Hydraulics). Most of the earlier experiments were made upon rectangular canals of comparatively small depths and of comparatively smooth beds—conditions not realized in the more important streams, and which, on account of the small dimensions of the cross section, rendered it impossible to determine by direct measurement the velocity at all points with sufficient accuracy to enable us to detect the true point of maximum velocity. It was in connection with experiments upon such a canal that M. Bazin, about the middle of the present century, concluded that the maximum velocity at the middle of a stream was at the surface, although he admitted that between the middle and the sides the maximum velocity might be a short distance below the surface (see Morin's Hydraulique).

Experiments upon rivers of considerable size have shown that the point of maximum velocity is below the surface of the stream*. Since the discovery of this principle, many writers attribute the loss of surface velocity to the frictional resistance between the water and the superincumbent air. But this assumption was disproved by Humphreys and Abbot, by finding that when the wind was blowing down stream with the same velocity as the surface of the stream, or even with a greater velocity, the point of maximum velocity was still at a perceptible distance below the surface. Boileau was satisfied from his experiments that the diminution of velocity from the

* Presented at the Eleventh Annual Convention, June 17, 1879. Read July 16th, 1879.

*See Ellett on the "Mississippi and Ohio Rivers," 1853; Boileau, "Traité de la Mesure des Eaux Courantes," 1854; Darcy et Bazin, "Recherches Hydrauliques," 1865; "Report on the Physics and Hydraulics of the Mississippi River," by Captain Humphreys and Lieut. Abbot, 1861; Morin's Hydraulique.

maximum to the surface could not be attributed to the frictional resistance of the air. He says: "We must seek for an explanation of this phenomenon principally in the mutual actions which bind the particles to each other, and in the oblique and rotary movements among the particles which result from these mutual actions and the unequal velocities of the neighboring particles." This statement, made a quarter of a century ago, contains sound philosophy, and furnishes a *general* explanation of several phenomena, the particular reasons for which it is not so easy to assign. For instance, it is not easy to explain why *particular* whorls are formed in water, or why they take a *particular* path, or why parts of a stream, in certain cases, flow up stream while the general declivity is down stream, &c.

The path of a particle in a stream of water of uniform *régime* is exceedingly tortuous. The experiments of Francis, cited at the beginning of this article, show that the particle leaves the bed of the stream and ascends more or less gradually to the surface, and as this may take place with all the particles, it necessarily follows that those at the surface must descend to the bed. We are not, however, dependent upon this experiment for establishing the fact. Any one, by observing a stream of say 5 or 6 feet or more in depth, can see that the surface appears to boil. This is caused by the ascent of bodies of water rising from some depth to the surface, and there spreading out upon the surrounding water. It thus appears that the particles, instead of moving among each other like filaments, go in bodies like streams of a few inches—or sometimes of a few feet—in diameter moving through the main body of the water. Or, possibly, these bodies are more like spheres, or other inclosed masses of water, passing from the bed of the stream to the surface, where they spread out, and afterwards form new combinations.

The motion here referred to is mostly in a vertical plane, but similar causes produce lateral movements. Every body projecting from the shore into the stream deflects the water, causing a wave-like motion extending from the shore to, or past, the middle of the stream. The retarding influence of the sides of the stream will cause a lateral action, just as the resistance of the bed of the stream causes a vertical action; and as the water along the shore goes more or less gradually towards the centre, it follows that the water along the centre must move laterally outward to supply the place of that which leaves the shore. It is hardly necessary to note the fact that the lateral movement is not in a plane, nor that its intensity is small compared with the vertical action, although it is an element not to be entirely ignored in considering the general problem. Since both the vertical and lateral actions are induced by the forward movement of the water in the stream, we might expect that there would be a greater accumulation of water where the velocity is greatest; and such is known to be the case, the water being highest at that point in the transverse section when the velocity is greatest. The cause of this elevation is not due to the lateral and vertical actions, but rather to the fact that the lateral pressure of a fluid diminishes as its velocity increases.

It is highly probable that the vertical movements originate at the bed of the stream, for any irregularities, such as small asperities due to a sandy or gravelly bed, or sticks or weeds, or changes in the slope of the bed, will cause the water either to ascend or descend. Since the density of the water is uniform throughout, it will, when once deflected upward in its course, continue to rise until it reaches the surface unless arrested by the downward movement of other bodies of water. We have reason to believe that this action is continually going on even where its presence is not easily detected by direct observation. It must, however, be kept in mind that these vertical and lateral movements are only secondary to the longitudinal one, and hence enter, not as a principal, but as a modifying cause. Granting these premises, it follows that the apparent, or measured, velocity at any point of a stream of water is the resultant of the volocities of many particles, and is not necessarily the actual velocity of any of them—some particles moving laterally in reference to the measuring apparatus, some slower and some faster than the measured velocity.

As any body of water is rising verti-

cally in the stream, its motion is retarded by gravity, and as it starts with a slow velocity at the bed it acts upon all the water through which it passes, tending to retard the velocity of the stream, while the stream by acting upon this mass tends to impart to it the same velocity as the stream. There is a tendency towards a common velocity between the stream and rising mass, but observation shows, as cited above, that, in many cases, the motion of each is, in a measure, distinct from the other. Such is particularly the case where the surface has the appearance of boiling. I do not now see how the ascending mass can account for the maximum velocity being below the surface. But the descending mass will have its motion accelerated by the action of gravity, as if there were no resistance, this acceleration would go on till the mass reached the bottom, like the motion of a projectile in a vacuum. But as the mass descends it comes in contact with water which is moving slower than itself, and hence its velocity will be retarded. But we notice particularly that for a certain fraction of the depth the velocity might—were there no vertical action—be *nearly* uniform, hence when the descending mass passes through it with a tendency to acceleration it will at first impart to the surrounding water an increased velocity, and this increase may be sufficient in many cases to cause the velocity below the surface to actually exceed the velocity at the surface. We cannot, however, at this stage of the theory, assert that this is the only, or even that it is the chief cause of surface diminution of velocity. It is possible that the lateral surface movement, caused by the action of the sides of the stream, as previously stated, is quite as important an element as the preceding.

Having indicated that the lateral and vertical movements may account for the point of maximum velocity being below the surface, we will suggest that a more critical study of the results of experiment and observation may confirm or modify this view. Thus, in a rapid, deep stream we would expect the point of maximum velocity to be deeper when the bed is rough and rocky than when it is smooth and regular. A study also of streams of different widths, but of the same depth and general character of bed, would enable us to judge more accurately of the effect of lateral movements.

These views also enable us to explain certain other phenomena. For instance, it has long been known that a boat of considerable dimensions will float down a stream more rapidly than the surface water. That this is the fact is known by observing that the boat floats faster than chips and small pieces of boards in the immediate vicinity of the boat. The latter objects being small will be resisted more in proportion to their weight than larger ones, and hence will move with nearly the same velocity as the water in which they float. Some have attempted to explain the comparatively rapid motion of the boat from the fact that it extends downward into the more rapidly moving elements; but this explanation does not apppear to be sufficient, for the velocity of the boat, in some cases at least, exceeds the maximum flow of the water. We submit the following explanation: The boat, in floating down stream, is partly in the condition of a body moving down an inclined plain, and, as such, may be considered as having a velocity independent of the stream. If we could conceive of the water in a stream having no velocity and offering no resistance (or only a very slight resistance) to the movement of the boat, the boat would (or at least might) slide down the plain. But in the actual case, the water does move, although it does not, as we have seen, go *directly* down stream. The boat slides, (so to speak) down stream, but the particles of water pass from the bed to the surface, and from the latter to the former; also from the side to centre, and from the centre outward, so that while the boat is taking a direct line down stream the water on which it is gliding is wandering about, taking comparatively long journeys.

We see from this that the *laminar* theory (so to speak) in which it is assumed that layers next to the bed of the stream remain in contact with it, and that the successive laminæ retain their relative positions is no longer tenable. Also that the supposed frictional resistances between consecutive surfaces will not account for all the varying velocities. Also that these lateral and vertical movements have an important bearing upon the phenomenon of "back water." Also

that much of the so-called "frictional resistances," or resistance due to the adhesion between the particles, may be explained by these movements.

WORKS OF SEWERAGE AND SEWAGE DISPOSAL.

By BALDWIN LATHAM, C. E., M. Inst. C. E.

From "Journal of the Society of Arts."

For some years past the Croydon Rural Sanitary Authority have had forcibly brought to their attention, the necessity for a complete system of sewerage and sewage disposal for Beddington, Merton, Merton Rush, Mitcham, Morden, and Wallington, being parishes or places under their jurisdiction for sanitary purposes.

The aggregate area comprised within these districts is 10,106 acres, or nearly sixteen square miles. The population at the present time is about 16,000, but it is very rapidly increasing, and since 1851 has more than doubled. Several centers of population are located in the drainage area, but at considerable distances apart.

Geologically, the district embraces the various formations found in the neighborhood of London, and lying over the chalk, which is the lowest geological formation in this district, and finishing with an alluvium overlying the London clay.

Naturally, the district drains to the Beverley Brook and the River Wandle, both of which streams flow in a northerly direction, and ultimately fall into the River Thames within the metropolis. The natural falls of the district are towards the metropolis, and as the district in question is contiguous thereto, it was a matter of some difficulty to provide an economical and sufficient scheme for effectually treating the sewage when a system of sewerage came to be carried out. It was considered that the cost in lifting the sewage, and carrying it a considerable distance in such a district, was quite out of the question, but ultimately a scheme was adopted, and is now in course of rapid execution, by which sixth–sevenths of the whole of the sewage of the combined districts will be conveyed by direct gravitation to land located partly in the parish of Wimbledon, outside the jurisdiction of the Croydon Rural Sanitary Authority, and partly in the parish of Mitcham, within the jurisdiction of the Rural Sanitary Authority.

An area of 38 acres of land has been purchased, by agreement, for which a sum of £11,700 was paid. Thirty acres of this land is an alluvial deposit, principally gravel, through the center of which formerly ran the old channel of the river Wandle, the remaining eight acres being stiff brick earth. In order to convey the sewage of the district to the proposed area, 36 miles of sewers are required to be constructed. Of these 36 miles of sewers, 29¼ miles are ordinary socket pipes, jointed with tarred gasket and Portland cement, varying in size from nine inches to 18 inches in diameter, and five miles of earththenware pipes with Stanford's patent joint. The latter also vary in size from nine inches to 18 inches in diameter. There will also be 1¾ miles of brick sewers, 24 inches, 27 inches, and four feet in diameter. The four feet sewer is to be used as a tank sewer in connection with that portion of the district, the sewage of which will not flow by direct gravitation on to the land, and it is made of the size in question in order to hold the sewage during the night, or at any period when the engines may require to be stopped. The sewage of one-seventh of the whole district, will require to be pumped to a height of 25 feet, including friction. It will then pass with the sewage from the gravitation area through a series of tanks, in which the solids will be deposited, and the liquid will then flow on to the area of land laid out for filtration. The land which is now being prepared for filtration is an alluvial bed 30 acres in extent, but as this land is entirely surrounded by rivers and streams, and is extremely porous, in order to fit it for

sewage purposes, it was deemed advisable to form all round it a puddle wall, which is carried through the alluvial bed of gravel and porous soil into the London clay lying below, and thus shut out the adjoining waters which have hitherto percolated into the land and made it a swamp. A main arterial drain is carried through the center of the land, and subsidiary drains will be made, by which the whole area within the puddle walls will be made the freest possible filter for purifying the sewage which is to be brought upon it.

The tanks for freeing the sewage from the solids are so arranged that chemicals may be used for producing precipitation before the application of the sewage to the land. It is not, however, contemplated to use chemicals directly to the sewage. Owing to the position of the works with reference to the contiguous population, and having regard to the desirability of preventing the slightest nuisance, the tanks for treating the sewage are placed under cover, and the solids or sludge, which will be removed by subsidence from the sewer, will at once be consolidated. For this purpose the sludge will be received in a lower tank than the filter tanks, where it will be treated with lime, so as to destroy its adhesive properties, and then be forced through filter presses, so as to consolidate it ready for immediate removal.

Experiments have been made with reference to the purifying properties of the area selected for filtration. Some portions of the soil of this area, in its natural state, are not so efficient as might be desired, when compared with other lands well known to be suitable for the filtration of sewage. There is no better way of testing land as to its fitness for sewage filtration than by ascertaining the percentage of water, by weight, it is capable of temporarily holding without loss. The most unfit parts of the area at Mitcham have been tested with the following results:

			Per cent. of water absorbed.
No. 1	Sample,	Yellow clay.....	53.3
" 2	"	Loam, sandy....	50.2
" 3	"	Peat.....	120.0
" 4	"	Loam.....	62.0
" 5	"	Loamy soil.....	60.0

If the above samples are compared with the characteristics of well-known lands adapted for the purification of sewage, it will be seen that they are greatly inferior to some of the following:

	Per cent. of water absorbed.
Surface soil of the Merthyr filters at Trœdyrhiw	39.82
Subsoil, do., do.....	45.77
Another sample of subsoil...	37.48
Additional land at Merthyr for the reception of the sewage of Aberdare—surface	49.05
Ditto, immediately below surface.....	41.35
Ditto, subsoil	13.07
Beddington sewage farm—surface.....	50.0
Norwood sewage farm—surface.....	59.0
Ditto, subsoil.....	41.0
Dursley soil (well-known for its purifying powers)	34.0
Chorlton Ees, near Manchester—surface.	56.71
Same soil after being burnt into ballast..	36.55

The porosity of soil, or the power to allow water to pass by filtration, is inversely in proportion to its capacity for retaining water. With rocks, however, it is the reverse, the smaller the percentage of water rocks will contain, the more dense and impenetrable they are to water. Clay soils, when burnt into ballast, differ in their physical properties very little from the Dursley soil, which was experimented upon by the Rivers Pollution Commissioners with such satisfactory results; and the experience of late years has shown that this material, when used as a filter, is well adapted for the purpose of purifying sewage; and so in districts where clay soil only is to be found, this material may be made into artificial filters, which will have a greater purifying power than is the case with almost any natural soil, perhaps with the exception of Dursley soil, which in its color and characteristics, is not unlike burnt clay.

The eight acres of clay land that have been acquired by the Croydon Rural Sanitary Authority, when made into artificial filters, can be made to effectually purify the sewage of about twenty thousand people. The unfavorable position which peat occupies in reference to its retention of water is confirmatory of the experiments of the Rivers Pollution Commissioners, who showed that it was about the worst material that could be selected for a filter, and in the case of the Croydon Rural Sanitary Authority's works, will require to be entirely re-

moved, and a more efficient material substituted.

The total cost of the Croydon Rural Sanitary Authority's works is estimated at £96,000. This amount, however, will no doubt be exceeded, as there are many new streets now being formed in the district, all of which it will be necessary to sewer. The cost to the present ratepayers of the district will be entirely covered by a rate of 15*d.* in the £, which includes the working expenses and the repayment of the principal and interest. No allowance is made for any returns from the land to which the sewage will be applied, and the rate includes the interest and repayment of the principal for the purchase of the land. There will, however, be a considerable return from the land, which cannot, in this neighborhood, so near to the metropolis, be put down under from £15 to £20 per acre as the value of the crops, which would not be all profit, as the cost of cultivation must be deducted; but even if the cost of cultivation equaled the value of the crops, it is true economy to utilize the sewage, when compared with the system adopted in the metropolis, and now extending to other places adjacent, of removing the nuisance from one spot to another lower down the same river. As the district of the Croydon Rural Sanitary Authority is fast increasing, both in population and rateable value, there is every reasonable probability of the rate being lessened. If the cost of these works is compared with the cost which has been incurred in adjoining districts, in which the sewage is disposed of by being collected into arterial drains, and then thrown into the Thames, it will be seen that the whole system of sewerage, with its works for purifying the sewage, will cost the ratepayers of the district but a tithe of the amount the ratepayers of adjoining districts have to pay for simply constructing the outfall sewer only. This is a strong example of the economy of locally treating and purifying sewage, rather than shifting a nuisance, at great cost, from one point to another, from which some day it will have to be removed at still greater cost.

THE HEAT EMITTED FROM THE HUMAN BODY AS AN ELEMENT TO BE REGARDED IN ARRANGING THE VENTILATION AND WARMING OF BUILDINGS.

From "The Builder."

A QUESTION of primary importance to the architect, and especially to the architect who is entrusted with the designs for any important public building, is that of the heating power of the inmates and occupants of such buildings; that is to say, of the amount of heat which, under ordinary circumstances, escapes from the average individual during any given period of time. It is obvious that no arrangements for either the ventilation or the warming of an important building can be even theoretically perfect in the absence of clear knowledge on this point.

A pound of coal of an average quality will give out, in the process of combustion, rather more than 14,000 British units of heat, each such unit being the amount of heat necessary to raise the temperature of one pound of water 1° Fahrenheit of heat. To effect this combustion, 140 cubic feet of atmospheric air, weighing 11 lbs., will be required, and the temperature of this quantity of air, if it were kept free from any disturbing or cooling influences, would be raised as much as 310° Fahr. by the combustion, or say from 62° to 372° Fahr. Of course, it never occurs that the exact quantity of air is thus applied; but a basis of comparison is thus obtained.

The heat liberated by the respiration of an average human being in twenty-four hours will be found to be very nearly equivalent to that which is liberated by the combustion of a pound of coal. 350 cubic feet of atmospheric air are stated by Professor Huxley to be the quantity that passes through the lungs of an ordinary individual in twenty-four

hours. This quantity, however, is independent of the unmeasured, though very important, amount of respiration that takes place through the skin. Mr. Bailey Denton calculates that an assemblage of 2,000 persons will, in two hours, give off as much carbon as is contained in a hundred-weight of coals, in addition to 17 gallons of water in the form of vapor. If the quantity of carbon here estimated as consumed in the process of respiration be compared with the prescribed amount of 10 ounces required by the military dietaries (especially when we remember that this combustion goes on more slowly during sleep than in the course of the day), it will be seen that Mr. Denton's estimate does not err on the side of excess. Thus we obtain rather more than 7,000 units of heat as due to the pulmonary consumption of carbon in twenty-four hours, or to the aëration of the arterial blood which takes place in the lungs.

As to the heat which has been produced in the chemical action indicated by the escape of watery vapor in the breath or from the skin, the calculation is more doubtful, owing to the fact that we are not aware of the form in which the elements of this vapor were taken into the animal system. Thus they might have entered as water, or again as elements of those hydro-carbons which we are in the habit of consuming, such as fat or as alcohol. The formation of some thirteen ounces of water from its constituent elements, oxygen and hydrogen, would, of course, produce an amount of heat greater than that which would be occasioned by the mere passage of an equal quantity of water through the system. In the former case some 5,700 units of heat would be produced. Taking into account the unmeasured degree of heat produced by the dermal respiration, it will thus be seen that the equivalent of a pound of coal is as close to a measure of the heating power of a man, in twenty-four hours, as can be at present pointed out. That is to say, that twenty-four persons in one hour would liberate an amount of heat equal to that which would be produced by the consumption of a pound of coal in that time.

Taking this convenient unit, it becomes possible to calculate with considerable exactitude the allowance that should be made in the construction and ventilation of ball-rooms, assembly-rooms, churches, and other public buildings, for the heating of the air by the natural warmth of the persons whom it is proposed to accommodate. We shall also be able to speak with considerable exactitude as to the difference of temperature that arises in small unventilated bed-rooms, and as to the practical cause of that horror of ventilation which we find so general amoug our cottage population.

We have before seen that the quantity of atmospheric air required for the combustion of a pound of coal is 140 cubic feet, while that which passes through the human lungs in twenty-four hours is more than double that quantity, being 350 cubic feet; the former quantity weighing, in round numbers, 11 lbs., the latter 27 lbs. The heat liberated we have taken roundly at 14,000 British thermal units. The coefficient of the specific heat of air is given by Mr. D. K. Clark, in his valuable "Manual" (p. 385), as 0.2377. These figures will be found to indicate an elevation of the temperature of 350 cubic feet of air by 88° Fahrenheit as the result of the entire combustion; and if this were regularly carried on for twenty-four hours, and if no heat were lost by radiation, conduction, or any other means, the temperature of that volume of air would steadily rise by 3.66° Fahrenheit per hour. If now we ascertain the degree of dilution which the size and the ventilation of the apartments allow to the expired air, we shall find the bases of our problem to be laid down. But we purposely avoid any attempt at exactitude; and we recommend those of our friends who make use of our units to form their own calculations. The approximate amount of 14,000 British units of heat may, we assume, be safely taken. The specific heat of atmospheric air is a figure easily to be verified. The volume of air through which the heat has to be distributed each investigator may calculate for himself.

We thus find that if we allow a sleeping apartment of the cubical contents of 1,000 cubic feet for two inmates, and suppose that there is absolutely no ventilation, in eight hours 9,333 British units of heat will have been emitted, and

will have gone to raise the temperature of a fraction over 80 lbs. of air. It will be seen at once that the resulting heat would be intolerable. Practically such a condition never occurs, because rooms are never really air-tight, and as the internal temperature rises, the escape of the heated air, and its replacement by the external atmosphere, becomes more and more active. But it will be seen how powerful a calorific agent is at work. And it will readily be understood how the desire for warmth, which is one of the most unfailing instincts of higher animal life, including that of man, naturally overpowers those sanitary conditions which appeal rather to the intelligence than to the feelings.

For the mighty influence of use and of habit must be borne in mind. We may take a case that is directly in point. Suppose a church to hold 500 persons, and that during a given service the doors have been shut, and the ventilation has been left to take care of itself. The effect on the temperature of the church, supposing the service to last an hour and a half, will have been equal to that produced by the combustion of rather more than 30 lbs. of coal within the building, no chimney being supposed to carry off any of the heat or carbonic acid gas generated. The actual rise of temperature will depend on the two elements of the cubical contents of the church and the activity of ventilation. The latter, it is true, may be so managed as properly to control both the vitiation and the heating of the air. But we are supposing the ventilation to be imperfect. To the persons within the church the inconvenience occasioned by the deterioration of the air they breathe will be very gradual, and will probably go on increasing for a considerable time, before any but those of them who are most delicately organized perceive it. When they quit the building the congregation at once become sensible of a difference in the external air as compared with that they are leaving. But the place must have been over-crowded or under-ventilated to a considerable extent if they become conscious of the fact before quitting their seats. If, on the other hand, we now suppose a fresh congregation to enter, as the first goes out, the degree of heating and of vitiation, which had so gradually been attained as to be almost imperceptible to the persons who caused it, will strike powerfully on the senses of the incomers. This is an experience which is common to almost every one. But it is not so common for persons to draw from it the true lesson. It is one to be worked out in numbers of attendants, cubic feet of building, degrees of the thermometer, and volume of ventilating movement. As such we commend it to the very careful attention of the architect.

If we now regard the mode in which the air, when thus heated by the caloric liberated from the inmates of a building, tends to find its own thermometrical lever, we shall be at once struck with the powerful agency that is at the disposal of the scientific builder.

The pressure at which it is usually calculated that gas for illuminative purposes should issue from the burner is stated to be that of half an inch of water. With this pressure the issue of the jet has a considerable degree of violence, as is more particularly to be noted in the case of fish-tail burners, where the spread of the flame is due to the pressure, and thus to the velocity, at which the gas issues from the orifice of the burner. Now a difference in pressure of more than half an inch of water exists between two atmospheric strata that are 50 ft. apart in vertical distance. If we consult any barometric table constructed for hypsometric purposes (that is to say, for indicating level above the sea by the reading of the barometer), we shall find that a fall of 1 in. of mercury roughly indicates a rise of some 900 ft. in altitude. We say roughly, not from absence of exact figures in the tables, but from the fact that there are two distinct causes of variation, one of which is easy to calculate, while the other is less reducible to rule. At the same spot, as is well known, the reading of the barometer varies from time to time. This variation constitutes the uncertain element. But taking the barometer to be steady, the difference in level indicated by a difference between two simultaneous readings occurs in a ratio dependent on the absolute level of those readings; that is to say, on the height of the stations above the sea. Thus the difference between the readings of 29 in. and 30 in. is tabulated at 883 ft.

But that between 29 in. and 28 in. is equal to 914 ft., and the fall from 28 in. to 27 in. of mercury gives a difference in level of 938 ft. Thus we have to regard both the ordinary local reading and the variable daily reading of the barometer with much care when we come to calculations as to levels.

Taking, however, the first tabular indication which we have cited, that of 1 in. of mercury indicating a rise or fall of 883 ft. in level, it will be found more convenient to reduce the vertical measurement to round numbers. On so doing, we find that a difference in level of 100 ft. will give a difference of 0.113 in. in the height of the mercurial column, or a difference of 1.260 in. in the column of a water barometer; so that 50 ft. difference of level may very well be taken, on an average, as equal to half an inch of barometric pressure measured by a column of water,—the relation of the 1.260 in. per 100 ft. varying as we approach the top or the bottom of the vertical distance.

Thus, the architect who intends to make arrangements for the ventilation of a large building,—whether he construct a shaft for that purpose, or whether, as in some of the noble common rooms at Oxford (we can take the dining-room of New College—if memory serves aright—for an example), the lower boards of a coved wooden ceiling are pierced, and the heated air, escaping through them, finds its way between the outer roof and the inner ceiling, to a louvred lantern,—has, in the first instance, this power of barometric pressure to aid him. That is to say, that in a building 50 ft. high there will be a less resistance to the issue of a column of air from an aperture at the top than from an equal aperture at the bottom (other things being alike), which is equal to that of a column of half an inch of water. This is only one part of the power at the command of the architect, but it is a very essential one, and one which it is highly important that he should be able accurately to measure.

The other element of power at his command is thermal. Barometric and thermic action may be made either to assist or to counteract one another. Too often the latter is the case. We cannot too much insist on the necessity for a perfect understanding on the part of the architect of the laws of these two causes of movement,—and that not only in general principle, but in numeric exactitude. We are, perhaps, apt to regard the efforts of scientific men to place useful knowledge in the hands of those who require it as dealing too much with what is elementary. But two things are, or may be, intended by that word. Elementary knowledge, to be real knowledge, must be exact. Exactitude is as requisite in the A B C of science as it is in any of the subsequent processes. But to have what is called a general idea,—which may be enough to allow a man to talk on a subject, but which altogether fails to guide him when questions of number, of measure, or of weight have to be decided,—is often spoken of as an elementary acquaintance with a subject. It is, in point of fact, an elementary ignorance.

Fifty feet of elevation, in round numbers, we have seen to give a barometric force equal to that indicated by the pressure of a column of half an inch of water. We now have to inquire what unit of heat may be taken as equivalent to, or as comparable with, this unit of barometric pressure. If more exact numbers are preferred, we have shown what corrections come into play to modify the rough rule of an inch of water pressure per 100 ft. of atmospheric pressure, that is to say, of vertical height.

The volume, density, and pressure of air at various temperatures is tabulated by Mr. D. K. Clark, at p. 351 of his "Manual for Mechanical Engineers." The density or weight of 1 cubic foot of atmospheric air, at a temperature of 32, is given at .080,728, which is the figure of which we have previously made use; but the corresponding barometric pressure is stated at 13.86 pounds per square inch, so that to bring this table into exact accordance with that from which we have previously quoted the relations of height to barometrical pressure, the former must be supposed to be based on observations made at the height of about 2,000 ft. above the level of the sea. We call attention to this fact as illustrating the manner in which the path of a scientific writer is beset by difficulties, and how inaccuracy may arise in the exact citation of the best authorities, if such

authorities are independent of one another. A pressure of 30 in. of mercury, which the barometric table assumes to be normal for the sea-level, is that which, according to the thermometric table, is equal to the pressure of a given weight of air having a constant volume at a temperature of 73° Fahrenheit. We shall not occupy our space with a reduction of the two tables to the same terms, but are satisfied with calling the attention of the reader to this slight element of discrepancy.

The rise of temperature from 62° to 70° Fahrenheit, according to Mr. D. K. Clark's table, is equivalent to an increase of pressure per square inch, from 14.70 to 14.92 lbs. This is equal to .0275 lbs. per square inch pressure, per degree of Fahrenheit, in increment or decrement. Taking the rough rule that a foot of water is equal to an inch of mercury in barometric balance, and allowing 30 in. of mercury to represent 15 lbs. per square inch of pressure (which will be found more useful than any more elaborate formula), we find the increase of a degree in the temperature of the air to be equal to .055 in. of mercury, or to .66 in. of water, as barometric pressure. Thus allowing for the escape of heat, we may take the Fahrenheit degree as the approximate equivalent of the half an inch of water, or of the 50 ft. of vertical elevation.

It is evident from these figures (of which we have indicated the authorities with the express intention of inducing our readers to make calculations for themselves) that the thermic power at the disposal of the architect is very much more readily available for the purposes of ventilation than the barometric power. At the same time, the essential importance of the latter is not easy to be overrated. It is an indispensable element to take into account. While impetus is given to the circulating volume by thermic force, the direction of the motion must be impressed on the circulation by barometric force, or by equivalent me chanical means. Here is the key to the structural question of the proper height of chimneys. If artificial means are asked for the production of a current of air, as by a fan, or other mechanical apparatus, the emission may be so far under the control of the architect that he may lead his flues where he will. But if he relies on the two natural motors of thermic and barometric pressure, woe will betide him if he neglects to give due attention to the latter. He may thus not only neutralize the action of the thermic force, but even set it to work in the wrong direction. It is the neglect of these primary principles which is the efficient cause of smoky chimneys.

Another element has to be regarded which may affect the movement of the ventilating circulation, especially when the barometric conditions are neglected. That is, the pressure of the wind on the apertures by which the vitiated air escapes. In this, as in the case of the thermic force, what is, properly regarded, a power at the disposal of the ventilator, is often allowed to act as an impediment instead of an aid. Thus, if a vertical shaft be constructed with an open and unprotected top, the effect of a strong current of wind is to keep back the escape of air from that tube; and so readily is this perceptible, that, in an ill-constructed chimney, blasts of wind without are often accompanied with puffs of smoke within the dwelling.

The pressure of the wind, according to Smeaton's experiments, rises from .005 lb. per square foot. with the hardly perceptible motion of a mile per hour, to 31.490 lbs. per square foot in a hurricane of eighty miles per hour. Other observers have estimated the force of a hurricane of 100 miles per hour, which Smeaton took at 49.2 lbs. per square foot, and later authorities as high as 62 lbs. per square foot. A pressure of 55 lbs. per square foot has been actually registered at the Glasgow observatory. Professor Airy has stated that a pressure of 80 lbs. per square foot may be obtained. The great storm of 1705 was estimated as traveling at the rate of forty-five miles per hour; and the resultant pressure was calculated by Smeaton at 9.963 lbs., and by later authorities as 10.1 lbs. per square foot of the surface exposed to its fury. This is equal to .07 lb. per square inch, or to .014 inch of mercury, or 1.68 inches of water in the barometric column.

Thus the effect of a storm of unusual violence, supposing it to be directly exerted in preventing the escape of a ventilating column of air, would be about equal to the barometric value of a chim-

ney of 170 ft. in height. This, however, is dealing with one element alone. We are supposing the movement to be so sluggish that the thermic force can be neglected. But it is that force which it is, usually speaking, most readily in the power of the ventilator to regulate. To a good volumetric discharge, driven by an elevated temperature, and directed by a lofty chimney, the proportionate check offered by even the fiercest hurricane is comparatively small. For it has to be measured, not only by the barometric height, but by the actual velocity of the column of escaping air. And as that velocity may be increased, almost *ad libitum*, by the application of artificial heat, it is clear that the architect has little to fear from the wind as an enemy to ventilation, if he knows what he is about. We may take another opportunity to show how he may make a violent wind, instead of an obstruction, an assistant, to ventilation.

It follows from the laws which we have attempted in some degree to elucidate, that in the hands of the wise master-builder those causes which frequently impede ventilation, or, at all events, lead to great inconvenience, may be converted into the very motor powers of a well-regulated ventilating current. This, of course, will by no means be new to many of our readers, more especially to those of them who are at home in the well-digested system of mine ventilation, on which the yield of our coal-fields so closely depends. The point which we venture to think novel, is the inquiry into the radiant heat produced by the process of animal respiration, as affording, in point of fact, an element of ventilating power. The estimate of that heating power which we have offered is, of course, approximate only. It is, however, founded on the best scientific data as yet attainable. And it is enough to afford an interesting means of comparison in the case of thermometric observations on buildings containing many persons, especially when they are ill-ventilated. In these, and in the opposite cases, much information may be collected by noting, in addition to the elements of cubic contents, number of persons, and flow of ventilating column, the actual temperature of the entering and of the issuing air. In an arrangement made for the purpose of this investigation, the architect would find a very valuable field of study. Meantime, the rough rule that from twenty-four persons assembled in a building there is liberated in an hour an amount of heat equal to that produced by the combustion of a pound of coal, is one that may afford food for very profitable meditation.

THE NEW THERMO-ELECTRIC LIGHT BATTERY.

From "Nature."

It appears that a difficulty which it has long been the ambition of practical electricians to overcome, has at last been solved by M. Clamond. According to his statement, published in *La Lumière Electrique*, which is confirmed by the Count du Moncel, M. Clamond has succeeded in producing the electric light by means of his new thermo-electric battery. M. Sudré has also just published his design for a powerful thermo-electric battery, but we do not know whether this system has yet been put to any practical trial, whereas that of M. Clamond is now in actual use for the purpose of lighting certain factories in Paris. Full details of either system have not yet come to hand, so that it is only possible to state the general results at present obtained.

That heat could be transformed into electrical energy, was first discovered by Seebeck in 1822, who found that an electric current was produced when the junction of two dissimilar metals was heated. Little use, however, was made of this discovery as a source of energy, owing to the feebleness of the current to which it gives rise, although it has been of great service since the time of Forbes and Melloni in the investigation of radiant heat. Prof. Bunsen, by the employment of different metals from those hitherto tried,

found that he could increase the strength of the current, and M. Marcus, of Vienna, using alloys instead of simple metals for the positive and negative element, reduced the cost, while increasing the power of the battery. From a thermo-electric battery constructed on his principle, and also from a modified form, devised by Wheatstone, a current sufficiently strong to produce brilliant sparks, decompose water, etc., was obtained. This was in 1865, and but little progress has, until now, been made in this branch of science, with the exception of the improved forms of thermo-pile devised by Noe and by Messrs. C. and L. Wray, although the utilization of heat—especially solar heat—for the production of electricity has long attracted the thoughts of many experimenters.

M. Clamond has for some time been at work upon the subject, and has so far succeeded that his thermo-electric battery has been employed since 1875 in M. Goupil's factories. These batteries are formed of iron, as the electro-positive element, and an alloy of antimony and zinc for the negative; they are soldered together and arranged in a circular form, which can be built up as high as may be desired. The junctions of the metals are heated in the interior, but the electromotive force being proportional to the difference of temperature between the two extremities of each bar, it was necessary to make the bars long if a strong current was desired, and then the results were less satisfactory, owing to the increased internal resistance, the melting of the metals where they were soldered, &c.

It is these hindrances to its extended use which M. Clamond has sought to obviate in his latest form of battery, which is composed of three distinct parts. The *collector* consists of a number of pieces of cast-iron so arranged that the heated air can circulate within them; a large surface is thus exposed to the heat, which the iron collects and communicates to the couples. The *diffuser* is the outside of the apparatus, and is made of sheets of metal. The *thermo-pile proper* is placed between these two, and is so arranged that the junctions of the metals are alternately at the temperature of the collector and the diffuser. Heat passes from the collector to the diffuser along these couples, which have no great length. In some forms which are very easily worked, a number of these couples are made into a flexible chain of any desired length, the extremities forming the poles of the battery. These chains, insulated from the other parts of the apparatus, can be united to each other by their free ends, so that a variety of couplings and combinations may be made. The model now in use for lighting a workshop in Paris is about 2½ meters high, and 1 meter in diameter, the exterior form being that of a polyhedron, to the sides of which the thermo-electric chains are attached: these are composed of small tubes of zinc and antimony joined together by plates of tin, to which they are soldered. Each half of the apparatus has 30 chains of 100 couples each, or 6,000 couples in all. To the outer surface of these chains are fixed the sheets of copper which form the diffuser or heat distributor.

Another model, made for the recent exhibition at the Albert Hall of the various sytems of electric lighting, is square and much smaller, though of the same power.

Each half of the cylindrical battery can be made to supply a powerful electric light, while the square one can produce *four* lights of half the brilliancy. The electro-motive force is, according to prolonged experiments, 218 volts, about equal to 120 Bunsen cells, while the resistance is 31 ohms. The large battery consumes only 9 or 10 kilogrammes of coke an hour, and the smaller one even less, about 6½ kilogrammes. Moreover, the large exterior surface of the apparatus radiating its heat to the air around adapts it admirably for use in heating, as well as for lighting, and it can thus be made to serve the double purpose of giving warmth and light.

M. Sudré has also designed his thermo-pile with a view to obtaining one of small volume, and having a low internal resistance; the other peculiarities of his battery consist in the manner in which one set of junctions are heated while the other set are cooled. He has also determined what is the best length for the bars forming the couples, in order that the necessary difference of temperature at the two extremities may be maintained, while yet making them as small as possible. This he finds should be from 10 to 30 millimeters, according to the difference

of temperature required. His manner of soldering together the two different metals is also novel and ingenious. In order that contact may be made with the whole surface of the bar, he cuts the plate, forming one metal into the shape of a comb, twisting the teeth of this comb together, thus retaining a large surface, which yet has only a short length. The bars are fastened on to these twisted parts and the uncut part of the plate is coated with silicate of soda. The couples are formed in a mould in which the plates are fixed, the melted alloy is then run into the mould so that a block is formed of the alloy and the plates, firmly united. These chains or blocks are then placed between two plates, coated on one side with enamel or other electrically insulating substance; several chains may thus be arranged side by side, each chain being both calorically and electrically insulated. The parts of the chain are electrically insulated by the thickness of the plates, but heat can flow across the couples. The chains are next placed between a collector and a diffuser; the collector is ribbed if the source of heat be gas, in order to expose a greater surface. The diffuser is also ribbed for the same reason when the heat is merely allowed to radiate into the air.

The whole battery is so arranged that the collectors form the inside of a circle, within which the heated air is circulated.

THE RUIN OF CANALS.

From "Iron."

I. THE CAUSES OF RUIN TO CANALS AND WORKS OF IRRIGATION.

In canals purely intended for navigation, the velocity of the water has to be kept below a fixed maximum, and below that may be anything down to still-waters without causing any harm; but in irrigation canals, which are continually receiving fresh supplies of water, and distributing it over the land through minor channels, the velocity of the water must be regulated with extreme nicety and care, in order to avoid many evils; the two extremes of which result either in making the canal utterly unremunerative from not carrying sufficient water for purposes of irrigation, or, in the eventual ruin and destruction, from deterioration, of the canal itself. Such canals cannot be maintained like roads, by merely repairing and trimming worn places; they also require that their suitable velocities should be perpetually watched and regulated, even in the case that the intended velocities were originally correctly determined, and the designs and works made in accordance with them.

One of the most important causes of ruin to works of irrigation is that the velocities were never originally well determined, but were faulty and unsuitable, if not throughout the whole of the works, then, at least, in portions of them, the result of which eventually affects the whole. This is the case with a great many Indian canals, and is likely to be so on many others, as the matter of hydraulic velocities is one on which knowledge is extremely small, for want of sufficient detailed record of trustworthy experiments.

The next cause in point of importance is faulty engineering, design and defective construction of the works themselves, but this admits of remedy, without going in most cases to such an enormous expense as the former class of error entails. Even under this head, the apportionment of the velocities at intakes, outlets, bridges, and such works, is of extreme importance.

Thirdly, even if we assume the comparatively unusual case of the original intended velocities and the works themselves having been correctly designed in the abstract, and of the works having been constructed to perfection, the canal itself may yet follow the steady course to ruin. For whenever ruin falls on the canal, or freshets or floods occur in any of the streams, rivers or sources of supply, which then increase the supply of the canal, the depth of water in the canal is increased at certain places; and be-

sides, the hydraulic gradient is increased, thus causing a very large increase of velocity taken in proportion to the adjustable correct limits. Under the same circumstances, too, a certain amount of silt is washed into the canal from its banks, and silt-bearing water may also, from want of early precautions, enter from the streams of supply. A high wind may also increase these evils. While, again, the velocity of the canal water may again be increased by the augmented velocity of the water entering the canal.

The practical adjustment of the velocity, or its regulation, becomes, under such circumstances, a matter of extreme care and refinement, even with the aid of all the hydraulic science the world now affords, and the assistance of good instruments and appliances for determining velocities; while without both of these aids it is nearly impossible in most instances.

Setting aside the extreme cases in which the excess of water admitted may be so large that it becomes necessary to let it out over the country by breaking down a bank, and assuming the very moderate one of the velocity being increased by only one-fifth, this alone is amply sufficient to cause scour and erosion of bed and banks to a very appreciable extent; and if this recurs at rainy seasons for years, it becomes positive ruin, not merely on account of the erosion itself, but because also the scoured matter is transported by the water in the form of silt and deposited at other parts of the canal. The whole regimen of the entire canal thus gets out of order, the velocities are re-distributed unsuitably or in ill-proportion; such errors augment very rapidly, and a partly worn and partly silted-up canal is the result. This is ruin, which cannot be set right except by extraordinary repairs costing half as much as the original cost of the canal; and this is the principal cause of ruin on works of the very best design.

Other causes of deterioration are the admission of silt-bearing water at intakes, neglect of petty repairs, and non-removal of such an average amount of sediment as may be deposited in the canal and channels from causes apart from the preceding. It may also be mentioned that neglect of repair in one year is not compensated for by double the amount in the next, under similar circumstances; but that all such results are cumulative, from increase of interference with the strict regimen of the canal, and its suitably apportioned velocities in various parts of its course.

The consideration of the causes, and more especially of the principal ones, leads to the inevitable conclusion that a careful adjustment, measurement and regulation of the velocities of the water in canals and works of irrigation is the basis of almost all measures for preventing or deferring eventual ruin.

That considerable refinement is necessary is evident from the fact that the maximum velocities in canals are:—

2.5	feet per second	for	very sandy soil.
2.75	"	"	sandy soil.
3.	"	"	loam.
4.	"	"	gravel and very firm soil.

While with low velocities of 1.5 and 1.75 feet per second, any suspended silt may be deposited, and vegetation springs up —the other source of extreme damage. The interval between the extremes is comparatively small and very easily overstepped.

Our present knowledge of velocities, their calculation, determination and measurement is extremely coarse at present (not long ago it was altogether erroneous), hence the necessity for more knowledge and greater refinement, which should be based on extremely careful experiments, carried out under the most advantageous circumstances, with all the aid that improved instruments and appliances of every sort can give and civilised assistance can furnish. The results may then, if correctly made use of and applied, prevent the lamentable ruin to our Indian canals which is illustrated by so many nearly obliterated ancient works both there and in other formerly well-irrigated countries.

II. ESTIMATED LOSSES FROM DETERIORATION OF IRRIGATION-CANALS.

The causes of deterioration, and the remedy for them, having been previously explained, the next point to be considered is whether it is worth while to go to the expense of investing money in experiments that will give the requisite more refined knowledge of hydraulic velocities, and of the methods of dealing with them. The amount actually invested in

India in canals and works of irrigation, including distribution done at all times, is certainly not less than twenty millions of capital, clear of all working expenses. (For figures in detail, see Hydraulic Manual and Statistics. Allen. 1875.)

Now in dealing with statistics of this description, for purposes of argument, it is absolutely necessary that no exceptional case, rates or figures should be used; this rule will therefore be rigidly adhered to, and instead of dealing with any special case of canal, a theoretical canal under conditions that average well among actual statistics will be dealt with. Let us suppose a completely developed irrigation canal to have cost one million pounds, the irrigated area to be half a million acres annually, and the net annual profit 10 per cent. on the capital. (The Eastern Jumna Canal yields 22, the Western Jumna Canal 31, and the Kalerun 24 per cent., and these are the three completely developed canals of India, while it is evident that half-developed canals do not afford a fair basis of calculation, any more than partly opened lines of railway). Now although the duration of a canal, or its lifetime, cannot be actually rigidly estimated, it is perfectly fair to assume that a canal relieved from the wear and tear of excessive velocities and from large deposits of silt, retrogression of levels, and so forth, which are all solely due to the causes previously explained, will last for a duration exceeding by a quarter the period that a less carefully managed canal will last; in other words, let us assume that if such a canal in one case will last fifty years, in the other it will only last forty years with the same prosperity, full average irrigated and full returns; while after that period they may steadily dwindle down from prosperity to ruin in a similar ratio. Taking the canal thus only at it climax, the total profits in either case will be in proportion to the number of years of duration; for the actual time when the 10 per cent. annual profit dwindles down to below zero, or the canal is worked at what is called a loss, is a different corresponding period in each case. Thus the compared profits on a capital of one million pounds will be about as follow:—

The difference of total profits, apart from either simple or compound interest on them, is about one million and a half

1. IN CASE OF MORE GRADUAL DETERIORATION.

10	per cent. for	50	years=	£5,000,000
8	"	10	"	800,000
6	"	10	"	600,000
4	"	10	"	400,000
2	"	10	"	200,000
1	"	10	"	100,000
Total profits during a century				£7,100,000

2. IN CASE OF MORE RAPID RUIN.

10	per cent. for	40	years=	£4,000,000
8	"	8	"	640,000
6	"	8	"	480,000
4	"	8	"	320,000
2	"	8	"	160,000
1	"	8	"	80,000
0	"	8	"	nil.
Loss during 12 years to be deducted at 1 per cent.				120,000
Total profits during a century				£5,560,000

pounds sterling, or half as much again as the original capital expended on one canal. Taking twenty such completed canals to represent the capital invested in India of twenty millions sterling, the loss due to the more rapid deterioration becomes thirty millions sterling, or half as much again as the capital invested; if extended over a full century in each case. Over half a century the loss is simply equal to the value of the capital invested, and this seems a probable and fair estimate of the anticipated loss in that period, or damage done.

To this estimated loss, or to something very near to it, there is only one alternative, and that is, the expenditure of the same amount in extraordinary repairs, which might be set down in the returns either as added to the capital account, or as included in the ordinary repairs. But however accounts may be managed, the amount estimated must either be lost, or spent in making head against the destruction occurring more rapidly in one case than in the other.

It is useless to ignore that there is a lifetime to everything; the principles of dilapidation cannot be controverted.

It may, however, be asserted that under any circumstances instructions may be given that the canals shall be kept in perfect repair, that every care shall be taken, and so forth. This is the very point; the care cannot be taken to prevent such damage unless a higher knowledge of velocities enables a more refined sort of care and prevention to be exer-

cised. No doubt the damage, instead of being allowed to accumulate over so many years into absolute ruin, may be stopped by incurring more expense annually; but this is merely spreading the bill for damage over a number of years; the expense is not prevented in that case, but merely divided; and if this form of account be preferred, instead of dealing with a total loss of twenty millions in fifty years, it becomes a waste loss, or combination of both, of £400,000 yearly over the whole of the irrigation canals and works of distribution of India, which is simply due to the coarseness of our knowledge about velocities. Comparing this annual waste, or even merely a quarter, or a tenth of it, with the relatively small cost of a thoroughly well conducted series of hydraulic experiments, we may easily see whether the latter are worth while, from a financial point of view, as a just and remunerative investment or expenditure on public works.

The principle involved cannot be avoided by drawing any analogy between canals and railways.

All improved modes and principles, and increased knowledge, experiments, and so forth, on railways, may have cost India nothing. As railways in their perfection were first required in England, where they are still being improved at the expense of skill, money and thought, all such ideas may be borrowed gratuitously. But there are no large irrigation canals in England, and India must necessarily work out its own improvements in that branch at its own expense, and effect permanent economies for itself, if at all; although it may, and perhaps should, bring to bear on them the highest English skill available in every respect, and make use of it both at home and in India.

In following up, or copying in practice, any clearly defined thoroughly-worked out principles, as those of roads, railways and navigable canals, a routine system of the marionette type may be sufficient for the purpose; but when practical improvement has to be gained by experience, experiment and skill, such a system is inapplicable without further aid.

The method hitherto adopted of following up and using the hydraulic experience and formulæ devised in France and Germany, and of applying their errors as well as their principles on a very magnified scale, thus saving expense in experiments, has had the most disastrous effect on the irrigation works of India; this point hardly requires exemplification. The isolation of Indian engineers, their want of opportunity, of reference to and consultation with English engineers in their special branches, is doubtless also to be deplored, as it accounts for many remediable defects; but the dispensing with extended special experiment and approved knowledge in a branch of science that pre-eminently affects the permanent benefit of the large and extensive works of irrigation of India seems perfectly indefensible either on financial or on any other grounds.

THE LIGHT AND ENERGY OF ELECTRICITY AND GAS.

By J. T. SPRAGUE.

From "The English Mechanic."

Some very remarkable statements were made before the Committee of Parliament on Lighting by Electricity. These statements are embodied in the report, and having gone the rounds of the Press without examination or comment, they will no doubt be adopted as settled facts. It is, therefore, very desirable that they should be examined, and that an understanding should be arrived at as to how far they are reliable, and in what sense they are true. We will, therefore, commence with the remarks of Sir William Thomson.

1,746. We are anxious to obtain your opinion upon various points connected with electric lighting; perhaps you will let me go to the root of the matter, and ask you to tell us the energy which is used and exhibited as light in the pro-

duction of the electric arc?—The energy which is actually used in the electric arc is about 1 horse-power per 2,400 candle-power, or even more than 2,400 candle-power, according to the dimensions and other circumstances of the electric arc.

1,747. That is to say, 1 horse-power, if fully employed, produces 2,400 candles or more?—Yes, 1 horse-power actually in the electric arc; 1 horse-power spent in stirring the luminiferous ether between the carbon points and on the surfaces of the carbon.

1,748. Have you at all calculated the energy appearing as light in the case of the combustion of gas?—1 horse-power of energy in the combustion of gas produces about 12 candle-power.

1,749. And 1 horse-power converted into electric light produces 2,400 candles?—Yes.

1,750. As these are rather startling figures, perhaps you will explain to the committee how you arrive at such results?—Sir William's reply is too lengthy for full quotation, as the greater part of it is unimportant; he concludes by saying, "The upshot of all is that, allowing the practical estimate of 1 horse-power actually spent in driving the engine to produce 1,200 candles, which has been realized, I estimate that one-half of that power goes to the electric arc, and one-half is lost in heating different parts of the machine. *This then gives* 2,400 *candles for* 1 *horse-power*, which is the figure of my answer. In respect to gas, I have taken 4 candles per cubic foot of gas per hour. I have taken the heating-power of gas at 12,000 centigrade units per gramme; I have taken the specific gravity of the gas as half that of air, and calculated accordingly. I have thus calculated the *whole heat of the combustion of gas*, from the amount of that per second of time, and reduced it to horse-power. Thus I arrive at the figure which I gave in my answer, *i.e.*, 1 horse power actually spent in warming the air and stirring the luminiferous ether in a gas-flame of 12 candle-power.

The committee in their report remark that there is some difference of opinion between the scientific and practical witnesses, and it is not to be wondered at if the foregoing is to be taken as a type of scientific evidence. In fact the statements are all true, but they are so arranged as to constitute a complete chain of special pleading, and to convey an absolutely false impression of what the facts themselves really mean. The statements are *true as to the energy actually in the lights.* The impression they produce *is false as to the energy necessary to produce the lights.*

There are two ways of considering energy, as with most other things. There is the *potential* or theoretical energy of a combustible; that is to say, the total heat capable of being produced by its union with oxygen. This potential energy of coal, for instance, is about 13,000 Centigrade heat units per lb. In commercial language, we may call this the *gross* energy of the coal. There is, again, the *practical* energy of the combustible, a function of the boilers and engines. For instance, average steam engines give a horse-power for 7 lbs. of coal per hour; this, in commercial language, may be called the *net* energy of the coal.

Now what Sir William Thomson has done is just this; he has charged the gas light with the gross energy, and the electric light with the net energy, as indicated by the italics in his evidence, and people who do not understand this, like the writers in the daily press, of course imagine that these figures so seriously stated are really true!

Now let us see what would come of an equitable comparison, using as far as possible Sir W. Thomson's own figures.

1 horse-power is 33,000 ft.-lbs. per minute, or 1,980,000 per hour. Expressed in heat units of 1 lb. water 1° Cent., which equals 1,390 ft.-lb., 1 horse-power per hour equals 1424.5. This, then, is the potential energy of the 4 cubic feet of gas, giving 12 candles light.

To put the 1 horse-power of energy into the electric arc, another 1 horse-power is of necessity expended in the circuit and must be charged, because the gas compared with it is the whole amount necessary. But to produce 1 horse-power in a steam-engine involves a consumption of coal varying from 2.5 lbs. in the most perfect and expensive engines known, to 8 and even 10 lbs. in common engines; let us take 4 lbs. as a very fair engine's work, and we have 8 lbs. coal expended in generating the 2,400 candles

mentioned. Now the potential energy of 1 lb. of average coal is 13,000 heat units, or in 8 lbs., 104,000.

Instead of having the energy per candle,

Gas .. 1,424.5 ÷ 12 = 118.7
Electricity 1,424.5 ÷ 2,400 = .597

This being what Sir W. Thomson makes it appear to be, we have by equitable comparison,

Gas .. 1,424.5 ÷ 12 = 118.7
Electricity 104,000 ÷ 2,400 = 43.3.

Even then we have omitted a very important element of consideration; the gaslight needs no machinery, no capital outlay, no skilled attendance; the electric light wants all these, and if they do not form an element of the scientific calculation, as to potential energy involved in the two cases, they enter very largely as factors in the practical calculation as to the potential cost. Hence the difference in the practical and so-called scientific opinions noted in the committee's report.

The comparisons made are, however, intrinsically erroneous, because made between electricity in its best conditions, on the large scale, and gas in its worst, on the small scale, and it is well known that in all things the small scale is most costly, whether we consider the expenditure of money or of energy. But, in addition to this general principle, the comparison is not really made with an electric arc having 1 horse-power of energy expended in it. The estimate is made from a light having 3 or 4 horse-power in it; in this way the value is arrived at "of 1 horse-power actually employed in driving the engine to produce 1,200 candles," and as only half this energy is in the arc, the light equal to 1 horse-power is taken as 2,400; but it is well known that to produce a light of 1,200 candles in a single arc requires 2 horse-power of engine, and in those conditions the 1 horse-power energy would be actually in the arc, and the efficiency would be

Electricity 104,000 ÷ 1,200 = 86.6

as against gas 118.7 per candle, and this when setting the production of a 12-candle light against one of 1,200.

It may be well to explain why the actual energy expended in the luminous area is so much smaller with electricity than with any combustible such as gas. In the case of combustion the energy has to be employed in heating the products of the combustion, steam and carbonic acid and a large bulk of neutral nitrogen from the air; these all flow rapidly away and carry off the heat. The very same loss has to be met in the electricity when derived from combustion, but the loss occurs, not in the light, but in the furnace and in the wires; here pure energy is collected and transmitted to the arc, where it makes its appearance as light unencumbered by the products of the processes of the transformation of latent energy into light and heat. Still, as the loss has to be incurred in both cases, both the lights, when compared, must have the total energy needed for their development taken into account.

The same witness expressed opinions as to the transmission of power from central engines to different parts of towns by means of electricity; but as Mr. Siemens gave the figures upon which these opinions are founded, we may now pass on to examine these. Mr. Siemens, comparing the cost of gas and electricity, says:—(136) "Making the unit of comparison 1,000 standard candles per hour, the results are striking. An Argand burner produced 1,000 standard candles of light per hour, with a consumption of 312.5 cubic feet of gas; and to make this, 56 lbs. of coal would have to be used in the retorts." (139.) "In the case of a medium dynamo-machine driven by a gas engine, 19.5 cubic feet of gas produce 1,000 standard candles during one hour; and the consumption of coal necessary to produce this gas is 3.26 lbs." (141.) "And in the case of the naked light, the consumption of coal is very nearly the same? It is very nearly the same: it is 13.3 lbs. in the gas-engine, and 12 lbs. in the steam-engine, showing that the gas-engine is really a very economical engine, and has moreover the advantage over the steam-engine, that no boiler is employed, and that the consumption of gas commences and ceases the moment the motion of the engine commences and ceases."

It will be seen presently that this advantage of the gas-engine is very much understated, because the coal corre-

sponding to the gas is largely over estimated.

The comparison between the relative weights of coal consumed is really of little importance, because the real thing to be considered is the relative costs. The cost of gas is, of course, greater than that of the mere coal it represents, because its price includes labor, capital, and distribution; on the other hand, the coal consumed in the other case must be charged with wages, &c., necessary for its consumption to place it on the same footing as the gas with which it is compared. But when any comparison is made, or any line of argument adopted, it ought to be correctly worked out.

Now, instead of an Argand burner giving the light of 1,000 candles on a consumption of 312.5 ft. of gas per hour, this value is calculated from the work of 62 Argand burners, each consuming only 5 ft. per hour of 16-candle gas. No burner of 1,000 candle-power has been made. But burners giving 250 candles on a consumption of 60 ft. are made, so that we know that 1,000 candles can be obtained from 240 ft., instead of 312.5. But Mr. Siemens commits a more serious injustice when he charges against the gas the whole of the coal put into the retort, allowing nothing for the coke and tar, which form part of the actual coal with which he is comparing the gas. In this way he obtains a consumption of 15 lbs. of coal per hour with gas as against 8 lbs. consumed in a steam-engine to develop the 2 horse-power necessary to develop 1,000 candles in one electric arc.

A ton of average coal gives 9,600 ft. of gas of .450 sp. gr., which equals 331 lbs. of purified gas per ton; about 1,300 lbs. of coke are produced, and if we reckon the excessive consumption of one-third of this in the furnaces, and charge the whole of this against the gas produced, we shall find that we get about 12 ft. of gas for each pound of coal which has disappeared—that is to say, 100 cubic feet of gas represent at most, 8 lbs. of coals. This reduces Mr. Siemens' 56 lbs. per 1,000 candles to 25 lbs. on the consumption of 5 ft. Argands, and to 19 lbs. on the actual consumption in 4 of the 250 candle burners.

This shows the great economy of the gas-engine as compared with the steam engine. Gas-engines are already in use which develop 1 horse-power per 21 ft. of gas, which represents at most 1.6 lbs. of coal, and this efficiency is realized in small engines. But the very best steam-engines of large dimensions have never realized more than 1 horse-power for 2.5 lbs. of coal, and this only with great attention. Ordinary engines consume 8 and 10 lbs. per hour per horse-power, and a consumption of 4 lbs. would be very good work for any engine employed under general conditions analogous to those for which gas-engines would be used. The reason of this economy is, that the potential energy of the fuel is converted directly into mechanical energy in the cylinder by the act of combustion, only so much is necessarily carried off as goes with the heated products containing only the small quantity of vapor due to the hydrogen in the gas. With steam, all this loss occurs in the furnace of the boiler, and in addition there is the immense quantity carried off in the escaping uncondensed steam.

We now come naturally to the question of transmitting power. Mr. Siemens says:—(200.) I believe it would be the cheapest mode of transmitting power to a considerable distance. The experiments which we have made give a result of a loss of 50 per cent., that is to say, 5 horse-power applied to a dynamo machine produces, roughly speaking, $2\frac{1}{2}$ horse-power at a distant point; it would follow that, say, 100 horse-power engine at a central station, which could be worked with a consumption of 2.5 lbs. per horse-power, would produce, at a number of points, power at the rate of 5 lbs. per horse-power.

Mr. Siemens thinks more favorable results may be hoped for than 50 per cent. return; but it is a pretty general law in electricity, that the resistances must be equal in generator and utilizer, and there are reasons for this, which the resistance expresses, though it is itself an agent and not the cause; hence 50 per cent. is the full theoretical efficiency, allowing nothing for loss in the conductor, and by leakage; in actual working the total efficiency would be more likely 30 to 40 per cent.

But giving electricity its utmost claim, and the use of the best engines known, we have a consumption of coal of 5 lbs.

per horse-power delivered, as against 1.6 lbs. per horse-power delivered by gas. But we have to consider something besides the theoretical consumption of coal. The 21 ft. of gas, at 4s. per 1,000, costs, 1d., while 1d. would buy 9.3 lbs. of coal at 20s. per ton. But the 1d. in the case of gas, covers the whole cost of the delivery of the energy of the coal to the machine which uses it. In the case of coal it represents, on the other hand, energy which has to be delivered; to place the electric energy of that coal on a par with the gas, we have to provide and pay for—

1. The capital outlay, wear and tear of the steam-engine, and its attendance.
2. The dynamo machine which generates the electric current and all its subsidiary expenses.
3. A conducting system, the conditions and difficulties of which are as yet absolutely untested.
4. The general expenses of management, and the profits of the undertaking.

These simply represent the gas-works and mains, and the expenses and profits of the gas company, all of which are included in the penny charged for the horse-power of gas. The consumer would have in either case to provide a gas-engine or dynamo-electric apparatus, the costs of which would be about the same.

It is quite certain that the expenses of all this would raise the cost of the 5 lbs. of coal consumed at the central station, to considerably more than the penny; in fact, they would be so great that it is not surprising that no one has yet ventured to dispute the remarks made some time ago on Mr. Siemens' proposal to transmit the energy of waterfalls 30 miles distant, or of the coal at the pit's mouth. The power costing nothing at the 30 miles distance, it would be better economy to allow to run to waste, its delivery would cost more than the expenses of carriage of coal, which has to be paid for, and its conversion into gas in the different towns as required.

As the writer is himself working at the problem of electric lighting, and is the patentee of machines which will be brought into operation in a little while, it will be understood that there can be no intention to depreciate the importance of these applications of electricity. But the statements examined will no doubt be utilized ere long, by the various prospectus writers, who are waiting their opportunity; and it is of great importance that their actual meaning should be understood, and the real truth be presented to the public understanding.

THE FOUNDATIONS OF THE NEW CAPITOL AT ALBANY, NEW YORK.

By WILLIAM JARVIS McALPINE, M. Inst. C.E.

From Selected Papers by the Institution of Civil Engineers.

The Author has not met with sufficient information to establish a rule for determining the load which may be safely imposed by a structure upon earth of a specific character and condition. The degrees of consistency and compactness in different kinds of earth, and their mixtures, and, above all, the extent of moisture therein, so affect the supporting power as to discourage any attempt at a formula for practical use. The nearly universal rule seems to be, to depend upon the previous experience of the locality, or upon observations of structures supported on similar earths in like conditions; in fact, to guess at, what the Author believes may, in most cases, be determined with considerable precision, and avoid on the one hand the unnecessarily costly foundations which are so frequently observed, and, on the other hand, those inappropriate and insufficient foundations which cause the destruction of the superstructure.

He was requested ten years ago, to devise plans for the foundations of a large and costly State building, which had to rest upon soil apparently equable

and stable, but which proved, on careful examination, to lack these qualities to a remarkable extent. It was also found that the earth, to a great depth under many portions of the foundations, received and parted with a considerable amount of moisture with the changing seasons. The circumstances of the case did not allow the use of piles or inverted arches; it was, therefore, necessary to spread the base of the walls over such an area as would afford the requisite sustaining power, and also to protect the clay and sand from any excess or deprivation of its natural degree of moisture, so as at all times to derive from it the same degree of support. The importance of the work warranted the expense of experiments to determine the questions above referred to. In the absence of any similar or equally extensive experiments, the Author is induced to submit the present ones, in the hope that an explanation of the methods adopted, and the results attained, will prove serviceable.

The structure, though a single building, may be considered as a collection of a dozen large ones, with great differences of elevations, and weights upon the lower walls, and yet so bonded together as to require that the pressure of each of the parts should be the same per square foot on the earth beneath. This object has been fully accomplished; and when the structure is loaded to the maximum extent of 200,000 American tons, the Author believes that it will not compress the earth upon which it rests more than three-fourths of an inch, and exactly the same under every part thereof. The building measures nearly 300 feet by 400 feet on plan, and has three main stories and a basement. The lower walls are 110 feet high, but those of the corner towers, pavilions and main tower, are of much greater height.

The ground covered by the structure sloped eastward at the rate of 1 in 25. The pit was excavated to a depth of 5 feet below the natural surface at the south-east corner, and 25 feet at the north-west corner. The excavation, together with the borings which were made in the bottom of the pit, fully exhibited the character of the earth. The lower strata (termed in the locality "blue clay" and "Albany clay") are more than 100 feet in thickness, resting upon the Hudson River argillite (a clay slate), the two forming the banks of the river for thirty miles of its course. The "blue clay" contains from sixty to ninety per cent. of alumina, the remainder is fine silicious sand. It also contains many nodules of clay, highly charged with carbonate of lime in the form of rings and discs about an inch in diameter. Overlying the blue clay was a mass of earth from 1 foot to 35 feet deep, composed of the same clay mixed with sand of different degrees of fineness, in proportions varying to such an extent as, when saturated, to render it in some places, a semi-fluid, while in others it was nearly pure sand, and very porous. This material occurred in veins and strata, large and small, above and below the level fixed for the foundation. One of the largest of these veins of viscid earth passed diagonally across the foundation, and at a depth of 6 to 20 feet below the bottom of the pit. It was 200 feet long, and from 5 to 25 feet wide.* Other veins and strata of less size, were found extending across the bottom, and sometimes terminating in pockets in the blue clay. Borings, from 10 to 30 feet deep, were made in several places below the bottom of the pit, which showed the substratum to be blue clay; and a well which had been sunk close by, to a depth of 100 feet, was entirely in the blue clay.

The earth in its natural condition at midsummer contained from 27 to 43 per cent. of moisture. When the samples were thoroughly dried and pulverized and again fully saturated (without dripping), they absorbed from 39 to 46 per cent. of water. The blue clay ordinarily held about 40 per cent., and when dried, again absorbed about 43 per cent. It was, therefore, as a rule, completely saturated in its natural state. It was upon this kind of earth that the subsequent experiments of the supporting power of the clay were made. The pure clay, obtained by separating it from the sand, weighed 116 lbs., and the sand so separated 80 lbs., per cubic foot; but, when they were again mixed in different proportions, the weight of the mixture was less than the proportionate means

* This vein was dug out, and replaced with clay and sand artificially mixed, moistened and slightly rammed in layers, so as to render it as similar to the adjacent natural material as possible.

between them. Earth taken from the same places as the samples, varied from 81.5 to 101.4 lbs. per cubic foot, depending upon the proportions of the clay and sand: and these weights show, to some extent, the relative supporting power of the earth at the places from which the samples were taken.

It was originally intended to support the structure upon wooden piles, of which a considerable number had been procured before the Author was entrusted with the direction of the work. Many comparatively large buildings in Albany have been supported upon wooden piles driven into the blue clay, or upon thick planks laid under the walls. In a few cases the wood used for this purpose has been found in tolerable preservation* half a century after it had been buried in the blue clay; but, generally, such timber was much decayed at the end of a quarter of a century; and several heavy buildings, after having stood firm for twenty years, began to settle, and the walls to crack, in consequence of the decay of the wooden supports, and the unequal settlements therefrom. It appears that when the clay had been kept constantly moist, the wood did not materially decay in half a century; but, wherever the moisture was drawn off, the wood did not last more than twelve years. In this case, even if a wooden foundation could have been arranged so as to be kept constantly wet, it would have ultimately decayed; and its use was, therefore, inadmissable. Cast-iron piles of white iron could be relied upon for a century or more, but would also have eventually decayed.

The use of sand and concrete piles, made by boring or driving holes into the clay and filling them with these materials, was also considered. For reasons which will subsequently appear, inverted arches could only be used under a part of the structure,† and it was deemed advisable to have but one system of support. The Author, therefore, finally determined upon the plan which has been executed.

In most buildings, except where spires or towers are introduced, the weight is nearly equably imposed upon the several foundation walls; but in the Capitol the main and pavilion towers are much higher and heavier than the adjacent walls. The extremely heavy fire-proof floors, loaded, as they will be frequently with dense crowds of people, books, &c., must necessarily carry their load to two only of the four surrounding walls, and, with some of the roofs acting in the same manner, will produce very unequal pressures upon the foundations.

The weight of the whole building and its contents when in use will be 200,000 American tons. The area of the base of the exterior and court walls, and the rear walls opposite, is about 24,000 square feet, and sustains an average of $6\frac{1}{2}$ tons per square foot on the basement walls. The main tower, which weighs 30,000 tons, has an area of 2,508 square feet, equal to 12 tons per square foot upon its foundation walls. The weight on the foundation under the exterior walls of the corner towers is 47 tons per lineal foot; on the interior walls of the same towers, it is only 39 tons; and on the adjacent division walls, $23\frac{1}{2}$ tons. Still greater differences in the weight on adjacent walls occur in other parts of the building, especially at the main tower, where the weight is 134 tons per lineal foot, and on the adjacent walls but 47 tons and 39 tons. Passing round the exterior walls of one quarter of the structure (the remainder being a repetition of the same sized walls), the weights to be supported per lineal foot are successively as follows: commencing at the main tower, 134 tons, (which may possibly be increased); the adjacent walls are 47 tons per lineal foot for 60 feet; next, $44\frac{1}{2}$ tons for 60 feet; next, 47 tons for 120 feet (turning the corner tower); next, $44\frac{1}{2}$ tons for 60 feet; next, 67 tons for 18 feet; and next, 50 tons for 52 feet to the center of the south or north front. On the rear of each of these walls, the interior wall is loaded with 39 tons, and and the division walls with $8\frac{1}{2}$ to $23\frac{1}{2}$ tons per lineal foot.

The exterior walls of cut granite facing, backed with rubble and brick, aver-

* In digging the trenches for the street pipes of the new waterworks at Albany, the Author had occasion to remove many of the old pine water-pipe logs, of which only the sap wood was decayed. They had been buried in blue clay for more than half a century.

† It was necessary to arrange to carry two-thirds of the weight upon the exterior, rear, and court walls, which are separated 120 feet on two of the fronts and only 90 feet on the other two. Inverted arches spanning three very unequal spaces would have imposed unequal loads upon the clay beneath, and their use would have defeated the design of distributing exactly the same weight upon every part of the clay beneath the structure.

age 150 lbs. per cubic foot. The floors, including the iron box girders, cross beams, brick arches and covering, average 24 lbs. per square foot. The possible weight of crowds of people upon the floors is taken at 100 lbs. per square foot; the snow upon the roofs, at two feet depth, is 12½ lbs.; and the effect of the strongest winds, which may at times be deflected perpendicularly against some of the roofs, is taken at 15 lbs. per square foot. The calculated weight which may come upon each of the walls is as follows: on the corner towers and front foundation walls, 47 tons per lineal foot; on the main east and west front, 50 tons; on the curtains, 44½ tons; on the ventilating tower, 67 tons; on the division walls, extending upwards through four stories, 23½ tons; on the partition walls of two stories, 13½ tons, and of those which extend one storey high 8½ tons, per lineal foot. The main tower is designed to be of stone, except the portion immediately below the dome, which, from being so high from view, was proposed to be made of iron. If it should be of stone to the dome, that change, together with some others, would increase its weight to 36,000 tons, equal to 14.4 tons per square foot at the base. Its footing stones were spread to 110 feet square, and the concrete to 125 feet square, and five feet thickness. The weight on the clay, with 30,000 tons, is 1.92 ton per square foot; and with 36,000 tons, it would be 2.3 tons; but it was arranged for an underpinning, if necessary.

THE EXPERIMENTS.

For the purpose of ascertaining the sustaining power of the blue clay in its natural condition, two sets of experiments were made; in the first by pressure upon a square foot, and in the second upon a square yard, of the surface. The machine used was a mast of timber 12 inches square, held perpendicularly by guys, with a cross frame for the weights. A hole was dug, 3 feet deep, in the bottom of the blue clay foundation, 18 inches square at the top, and 14 inches at the bottom. The foot of the machine was placed in this hole, and weights from 2,754 lbs. to 23,784 lbs. were applied. Small stakes were driven into the ground, in radial lines from the center of the hole, and the tops carefully driven to the same level; and by means of a straight-edge any change in the surface of the ground adjacent to the hole could readily be detected and measured.

Table I shows a continued settlement of the clay under the foot of the machine as the loads were added, but no change in the surface of the adjacent ground was observed until an hour after a weight of 11,844 lbs. had been applied, when an uplift of the surrounding earth was noted, in the form of a ring with an irregular rounded surface, the content of which, above the previous surface, measured 0.09 cubic foot, which is equivalent to a displacement of 1.09 inch of clay in depth under the foot of the machine, or equal to one-fifth of the whole settlement which had then occurred.

(*See Table on following page.*)

When the weight had reached 20,954 lbs., and had rested for half an hour upon the clay, a further protrusion was noted. The form of the ring was the same as before, but with more irregularity of surface. The highest part of the protrusion was from 12 to 15 inches from the edge of the pit, where it averaged 0.3 inch high, and sloped off outwardly to an average of 4 feet from the center of the hole. This uplifted earth measured 0.606 cubic foot, which is equivalent to a displacement 7.272 inches. When a weight of 23,784 lbs. had been applied, and had rested three hours on the clay, the ring in the highest part averaged 0.5 inch high, in the same general form and extent as before noted. The amount of earth thus raised was 1.01 cubic foot, equivalent to a displacement of 12.12 inches under the machine.

Before the lifting of the earth surrounding the machine could have taken place, the materials first displaced from under the machine were doubtless forced among the particles of the earth adjacent to the hole, and compressed that earth to some extent; and this operation was continued until the adjacent earth had become so compacted as to cause the lifts noted in the table. The Author is of opinion that the compression of the earth below the bottom of the machine continued without any considerable displacement until after a load of 4,000 lbs. or 5,000 lbs. had been applied, and that then the displaced earth found space in the adjoining earth until the load reached

TABLE I.

Observation.	Day.	Hour.	Duration.	Weights.		Settlement.		
				Each.	Total.	Each.	Total.	
			Hours.	lbs.	lbs.	Inches.	Inches.	
1	Mon.	5 P.M.	—	2,754	2,754	0.288	0.288	The weight of the machine.*
2	Tues.	9 A.M.	16	—	—	0.612	0.900	
—	"	9 "	—	2,820	5,574	—	—	First stone added.
3	"	10 A.M.	1	—	—	0.528	1.428	
—	"	10 "	—	6,260	11,834	—	—	Second and third stones added.
4	"	11¾ "	1¾	—	—	4.752	6.180	Uplift noted.
—	"	11¾ "	—	3,250	15,084	—	—	Fourth stone added.
5	"	¼P.M.	¾	—	—	3.588	9.768	
6	"	1 "	½	—	—	1.728	11.496	
—	"	1 "	5 min.	2,890	17,974	0.060	11.556	Fifth stone. Immediate settlement.
7	"	1½ "	½	—	—	3.288	14.844	
—	"	1½ "	—	2,980	20,954	—	—	Sixth stone added.
8	"	2 "	½	—	—	4.128	18.972	Uplift noted.
—	"	2 "	—	2,830	23,784	—	—	Sev'th stone added.
9	"	2½ "	½	—	—	5.184	24.156	
10	"	3 "	½	—	—	2.060	26.216	
11	"	4⅓ "	1⅓	—	—	1.300	27.516	
12	"	5 "	⅔	—	—	3.084	30.600	Uplift noted.
13	Wed.	5 A.M.	12	—	—	0.600	31.200	
14	"	8 "	3	—	—	—	31.200	No settlement after 5 A.M.

Observations 12 and 13 are not reliable.

* The first settlement, noted in observation 1, was due to the weight of the machine, and was not a compression, but only a leveling of the rough inequalities of the clay. The subsequent observation 2, of 0.612 inch, is the compression due to 2,754 lbs. This settlement occurred before 5 A.M.

7,000 or 8,000. lbs., when the uplift became visible at the surface of the ground; but that meanwhile the earth directly under the machine was continually more and more compressed in some proportion to the weight added. The small area pressed upon facilitated the escape of the material into the adjacent earth, which weighed only 300 or 400 lbs. per square foot. If the pit had been deeper, or the piston larger, there would have been less displacement.

The second set of experiments was made with the same machine, to the bottom of which was framed a strong base 3 feet square. The pit was sunk 2 feet deep into similar earth, and was 38 inches square both at the top and at the bottom. The stones were put on at intervals of an hour. There was no uplifting of the surrounding earth.

Table II shows a remarkable regularity in the settlement as the load was increased, and a constant diminution of the increment as the earth became more compacted. At the 6th observation, the weight per square foot corresponded nearly with the 2nd in table I, and the settlement was almost the same. The base was nine times as large, so that the proportion of escapement of the earth from beneath must at this time have been very small. It is probable, however, that if the weights per square foot had been increased so as to equal those in table I., a similar uplift would have occurred, though of less extent. The Author derived from the tables the opinion that the extreme supporting power of this earth was less than 6 tons per square foot, and that the load which might be safely imposed upon the clay was 2 tons per square foot.

(*See Table on following page.*)

For the purpose of maintaining the clay beneath the structure in the same condition of moisture, a deep puddle wall was extended entirely around the foundation, not only to exclude an excess of water, which might reach it through the veins and films of sand with considerable hydrostatic head, but also to prevent the egress of the natural moisture through similar veins. Although the puddle wall

TABLE II.

Observation.	Weight.			Settlement.		
	Each.	Total.	Pr. sq. ft.	Each.	Total.	
	lbs.	lbs.	lbs.	Inch.	Inch.	
Machine placed	3,228	3,228	359	—	—	No settlement that could be measured.
1st stone added.....	2,380	5,608	623	0.050	0.050	
2d " "	3,300	8,908	990	0.150	0.200	Estimated.
3d and 4th stone added...	6,960	15,868	1,763	0.166	0.366	
5th " " ...	2,830	18,698	2,078	0.134	0.500	"
6th " " ...	3,250	21,948	2,439	0.124	0.624	
7th " " ...	4,420	26,368	2,929	0.096	0.720	"
8th " " ...	1,190	27,558	3,062	0.080	0.800	
9th " " ...	2,320	29,878	3,320	0.025	0.825	

The notations of the settlement were generally made about an hour after the weight had been applied.

was carried up to the level of the terrace which surrounds the building, yet water might find its way along the face and down the outside of the walls, or possibly through some accidental break in the concrete floors within, and surcharge the clay below. To prevent this, there was spread on the top of the clay, over the whole area enclosed, a depth of 6 inches of course, screened gravel, the effect of which will be that under the great weight of the building, any excess of water in the clay beneath will be forced into this pervious gravel, and flow off through it to the drains which encircle and traverse the foundations. The necessity for these provisions will be apparent, when it is considered that many of the veins of sand extend to the surface of grounds of much greater elevation than the foundations, and that they communicate with imperfectly built street sewers and water pipes; while the same or other porous veins extend beneath the surface to grounds which are much lower. Through these sources the clay under some portions of the structure might be charged with water, while that under an adjacent wall might at the same time be drained of much of its natural moisture, and thus entirely destroy the design of a foundation which should everywhere have an equal sustaining power. It is not an absolute settlement which is to be apprehended, but a greater yielding in one place than in another.

A common practice of builders who have occasion to erect high and comparatively heavy towers and spires, is to groove the lower part into the adjacent walls, so as to allow the heavier ones to slide in these grooves, without breaking the bonding stones. In the present case, the demands of the architect forbad the use of grooving, and hence the necessity for the above provisions.

The main walls of the building are from 5 to 7 feet thick, where they rest upon the foundation walls, and bring upon them pressures of from 6 to 9 tons per square foot, which had to be reduced to 2 tons per square foot on the clay. This was accomplished by projecting each of the footing courses beyond those immediately above them. The rule was to commence with a load of 2 tons per square foot upon the clay, 3 tons upon the top of the concrete, and generally 4, 5, 6, or 7 tons upon each succeeding course of stone. The weight on each lineal foot of the top of the foundation walls, divided by the above pressures, gives the exact width of each course of the footing stones as shown in Table III.

(*See Table on following page.*)

The large quantity of stone required in a short time—50,000 tons in four months—compelled a resort to a great many quarries, which furnished stones of different thicknesses, and made it necessary to modify the above exact arrangement; but the principal of the distribution of the load, according to the vertical strength of the stone used, was maintained throughout the foundations.

It was necessary to consider how far

TABLE III.

Parts of the Building.	Load per Lineal Foot.	Required width of Course.										Main Walls.	
		Concrete.		Courses of Footings.									
				1st.		2d.		3d.		4th.		5th.	
	Tons.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.
Corner towers, front.	47	23	6	15	8	11	9	9	4	7	10	7	4	7	0
" " rear..	39	19	6	13	0	9	9	7	10	6	6	5	6	5	0
Curtains, front.....	44½	22	3	14	8	11	1	8	11	7	5	6	5	6	5
Central fronts.....	50	25	0	16	8	12	6	10	0	8	4	7	2	6	5
Partitions, 4 stories..	23½	11	9	7	10	5	11	4	9	3	11	3	5	3	0
" 2 " ..	13½	6	9	4	6	3	5	3	0	—		—		3	0
" 1 " ..	8½	4	3	2	5	2	2	2	0	—		—		2	0

these projections could be made without danger of breakage of the projecting part of the stone. The pressure in this case tending to break the stone is that due to the weight on the wall above it, divided by the width of the wall and multiplied by the area of the projection, and to treat that result as a load distributed on a beam supported on one end.

To distribute the weight upon the footing-stone courses with certainty, the beds of the limestone and granite were dressed to close parallel joints, so that the weight of each of the upper courses should be carried out to the extremity of the next course below. The vertical joints were only required to be quarry joints, not exceeding 1 inch wide. For certainty and convenience of laying the masonry, the foundation stones were all required to be rectangular blocks, of from 18 to 24 inches in thickness, the breadth to be at least one and a-half time the thickness, and the length two and a-half times the thickness. The average size of all the stones was 31 cubic feet, equal to 2½ tons, and many of them were from 5 to 8 tons weight. In the foundations of the main tower, the average weight of the granite blocks was 4 tons, and of the projecting blocks 7 tons. The footing courses were spread out equi-distant from the lines of the center of gravity of the imposed weight above. The exterior stones of the three lower footing courses were all headers from 4½ to 7 feet in length. The longitudinal bonding was made by the interior stone; and in the upper courses, where the projections were smaller, by alternate headers and stretchers of the front stone, as well as the interior. The result of this bonding will be to distribute the weight and equalize its pressure upon the clay.

The weight of the main tower was so much greater than that of the other walls, and the earth below it so much inferior, that the foundation was placed 7 feet deeper than elsewhere. With this exception, all the walls was commenced at the same level. The spaces between the main exterior, rear and division walls, and under the arches of the central court, were covered with a layer of concrete, made of screened gravel and hydraulic cement, 1 foot to 2 feet thick.

RESPECTING the East River Bridge the *North American* indulges in the following:—"The East River Bridge is likely to remain in the position of Mahomet's coffin for some time to come, according to present appearances. John Kelly will not permit any more money to go out of the city treasury towards its completion if he can help it, and now the city of Brooklyn, which has been paying its proportion without any delay, has decided not to spend another cent until New York pays up her share." Apparently the individual without reference to whom so many of the "London letters" of our country contemporaries could not be written, has also a contemplative scene to perform in America, for the writer concludes, "Can it be that the solitary New Zealander will still find a seat waiting for him on the piers?"

—*Engineer.*

THE BRUSH SYSTEM OF ELECTRIC LIGHTING.

By CHARLES F. BRUSH, M.E

Contributed to VAN NOSTRAND'S MAGAZINE.

PECULIAR FEATURES OF THE DYNAMO-ELECTRIC MACHINE.

THE most prominent peculiarities of the Brush dynamo-electric machine are embodied in the armature, the arrangement of field magnets, and the commutator.

The armature consists of a flat ring of soft cast-iron, revolving in its own plane. This ring is composed of two or more parts, each provided with a series of grooves, and insulated from each other, all in such a manner as effectually to prevent the induction of currents in the iron itself when it is revolved in a magnetic field. On this ring are wound eight bobbins of armature conductor, whose planes radiate from the axis of rotation.

The field magnets of the machine face both sides of the armature, in the plane of its rotation; thus both sides of the flat bobbins of armature conductor are exposed to the direct inductive influence of the magnets. This arrangement of armature and magnets differs radically from that adopted in the Gramme machine, the only other dynamo-electric machine of note in which an annular armature is employed.

Field magnets of opposite polarity are applied to diametrically opposite points of the annular armature, which then consists practically of two semi-circular magnets, having their like poles joined. Each of these semi-circular armature magnets, if straightened out, would be much longer than its own diameter, and would have its bobbins wound at right angles to its axis, and covering the greater part of its length. Now it will be evident, that with a given length and weight of armature conductor, a very much greater number of convolutions may be formed on such an armature, than would be possible on other armatures whose length, from pole to pole, is short compared with their diameter. But inasmuch as the field magnets are applied to both sides and very nearly the whole length of the long armature, the magnetic field in which the bobbins of conductor move, is quite as intense as that obtainable when any other form of armature is employed, and perhaps more so; hence the electro-motive force of current obtainable with an armature conductor of given resistance is, *ceteris paribus*, very much greater with this form of armature than with any other. For a practical demonstration of this let us consider the Brush machine known as size number 7. In this machine the resistance of the armature conductor, measured through the brushes on the commutator (the resistance is the same in all positions of the commutator), is about four ohms; in some machines a trifle more, in others a trifle less, according to the purity of the copper wire employed. When the armature is rotated at its normal speed of seven hundred and fifty revolutions per minute, in its normal magnetic field, the electro-motive force developed in its conductor is sufficient to maintain its normal volume of current through an external resistance, including the field magnets of the machine, of fully eighty ohms; or through a resistance twenty times as great as its own. No other form of dynamo-electric machine has yet shown a result comparable with this.

This current operates from sixteen to eighteen powerful electric lights arranged in a *single circuit*, giving to each an arc of about two millimeters length. The resistance of these arcs averages about four and a-half ohms each, as shown by many careful measurements of different lamps, both singly and in groups. (The measurements included the magnet helix of each lamp, the resistance of which is, however, quite small and will be given further on.)

The bobbins of wire on the armature of the Brush machine are not connected together in a single circuit, but each pair of diametrically opposite bobbins only, are connected together. The two free ends of the conductor, thus formed of each pair of bobbins, are carried to the commutator and attached to diametrically opposite segments thereon, which

segments are not connected with any other bobbins. Thus each pair of bobbins is entirely independent of any other pair.

In this arrangment, which secures important advantages, the Brush machine again differs radically from the Gramme, and all similar continuous-current machines.

The commutator consists of four separate rings of metal, each ring consisting of two nearly semi-circular segments, whose ends, on one side, are separated by a considerable space. This space is occupied by a piece of metal attached to an adjoining ring, and known as an "insulator." It is insulated by an air space from each of the segments between whose ends it is located, the other ends of the segments being simply separated by a single air space. The office of the "insulator" is to separate either of the brushes, which collect the currents from the commutator, from both segments during a certain interval, and twice in each revolution of the commutator. During these separations, the two bobbins of wire on the armature which are connected with the pair of segments, are not only out of the general circuit, but are open-circuited themselves, so that no current can circulate in them.

Each pair of bobbins on the armature is thus provided with a commutator ring, and the segments of this ring occupy the same angular position with regard to other segments, that its bobbins occupy with regard to bobbins attached to the said other segments.

It will now be seen that only one pair of armature bobbins is out of the general circuit at one time; and this is made to occur when the said bobbins are at and near the neutral points of the armature, and are not in a condition to contribute to the general current. Now, if it were not for the "insulators" above described, the idle bobbins would afford an easy passage for the current from the active bobbins, and thus destroy the efficiency of the apparatus.

The Gramme form of commutator, which is employed for the production of continuous currents in all machines other than the Brush, involves serious disadvantages, from which the Brush form of commutator is free. The principal difficulty arising from the use of the former is this:—The bobbins of wire, when at and near the neutral points of the armature, contribute little or no useful effect; but the current from the other bobbins must pass through them in order to reach the brushes, thus experiencing a considerable and entirely useless resistance, and, owing to the opposite directions of the currents through the active bobbins on opposite sides of the neutral points, these currents by passing through the idle bobbins, tend strongly to produce "consequent" points in the armature where the neutral points should be; thus interfering seriously with the proper distribution of the armature's magnetism.

It may be argued that the evil above indicated is eliminated by allowing each of the brushes to embrace several sections of the commutator, corresponding to bobbins on both sides of each neutral point; this is no doubt true to some extent, but another, and perhaps worse evil is thus introduced; the bobbins corresponding to the commutator sections embraced by each brush, are short-circuted in themselves, through the brush, and, owing to their comparatively small resistance, powerful currents are developed in them. These currents absorb much motive power in their production, and rapidly heat the bobbins.

Again, during each revolution of the Gramme commutator, each brush must break and make again, as many contacts as there are sections of the former, and each act involves the whole current; while with the Brush commutator, only two contacts and breaks occur with each brush in each revolution, and these successively in the different rings, so that but one-fourth of the whole current is involved at one time.

Further, the use of oil for lubrication, appears to be impracticable with the Gramme form of commutator, while it may always be employed on the Brush commutator with great advantage. Thus the durability of this commutator is made immensely greater than that of the other.

When necessary, the wearing segments of the commutator may be replaced by duplicate pieces in a very few minutes by any mechanic.

FEATURES OF THE LAMPS.

These are: very great simplicity of construction; ensuring ease of management, safety against internal derangement, and securing great regularity of working:—the double-magnet circuit conveying currents of opposite direction, by means of which any number of lamps may be operated in a single circuit without any irregularity of action:—the short-circuiting safety attachment, by which any lamp offering an abnormally great resistance, owing to the final consumption of its carbon, or other cause, will, without any change of strength in the main current, automatically short-circuit the said lamp, and thus preserve the integrity of the general circuit:—the multiple sets of carbons, burning successively, without the intervention of any switching or other special mechanism, and yet securing the maintainance of the light for any desirable length of time without requiring attention; lastly, the absence of any adjustment to be made by the attendant, other than placing the carbons.

These lamps contain no clock-work or similar mechanism of any kind. The movement of the upper carbon, actuated by gravity, is controlled by a simple annular clamp which surrounds the rod carrying the carbon. When the lamp is in operation, one side of this clamp is lifted by magnetic action; this causes it to grasp and raise the rod, and thus separate the carbons. As the carbons burn away, the magnetic action diminishes, and the clamp and rod move gradually downward, maintaining only a proper separation of the carbons; but when the tilted annular clamp finally touches the supporting floor from which it started, any further downward movement will at once release the rod and allow it to slide through the clamp, until the latter is again brought into action by the increased magnetism due to the shortened arc between the carbons. In continued operation, the normal position of the clamp is in contact with its lower support, the office of the controlling magnet being to regulate the sliding of the rod through it. If, however, the rod accidentally slides too far, it will instantly and automatically be raised again, as at first, and the carbon points thus maintained in proper relation to each other.

Each magnet helix is first wound with a few layers of coarse wire, through which the main portion of the current operating the lamp passes. Over this coarse wire is wound a very much greater length of fine wire, having its ends connected with the terminals of the lamp, but in such a manner that the electric current shall pass through it in a direction opposite to that in the coarse-wire helix. Thus the fine wire forms a secondary circuit of high resistance through the lamp, which circuit is independent of the arc between the carbons, and is always closed. It follows from the difference in direction of the current in the two helices, that the fine-wire helix will constantly tend to neutralize the magnetism produced by the coarse-wire or principal helix. The number of convolutions of the fine-wire helix and its resistance, are so proportioned to the number of convolutions in the principal helix, and its resistance together with that of the normal voltaic arc, that the magnetizing power of the latter shall be much greater than that of the former. Notwithstanding the small amount of current which passes through the fine-wire helix (about one per cent. of the whole current), its magnetic power is very considerable, owing to its great number of convolutions.

Now when a number of regulators provided with these double helices are operated in a single circuit, great uniformity of action will be maintained owing to the peculiar function of the secondary helix. Thus, when any lamp gains more than its normal arc, the resistance of its main circuit is thereby increased; more current is consequently shunted through its secondary helix, and the resultant magnetism is diminished, allowing the carbons to approach. On the other hand, if an arc becomes too short, its resistance is reduced, and less current is shunted through the corresponding secondary helix; consequently the working magnetism in that lamp is increased, and its carbons are drawn further apart. Thus it will be seen that, although the general strength of the current operating a large number of these lamps does not vary, each lamp performs its regulating functions through the agency of varying magnetism, precisely as though it were the only lamp being operated.

In practice, the resistance of the fine-wire helix or helices in each lamp is rather more than four hundred and fifty ohms; while the resistance of the coarse wire, various connections, carbons, and voltaic arc, in each lamp used with the sixteen-light machine, is about four and a-half ohms. Hence not more than one per cent. of the whole current is diverted from the arc. The resistance of the coarse-wire helix, carbons (copper-coated), connections, etc., in each lamp is very small. To determine this resistance, sixteen lamps were connected in series in the usual manner, about two hundred feet of number ten copper circuit wire being used. Full length carbons were then placed in the lamps, and the upper and lower carbon of each lamp were connected by means of a strip of sheet copper wired to each carbon. The resistance of the whole set was then measured, and found to be 2.10 ohms, showing a resistance for each lamp with its carbon, of .131 ohm. This is 2.91 per cent. of the whole resistance of the lamp when in operation. To this loss must be added the one per cent. due to that amount of current diverted from the arc by the fine-wire regulating helix, making a total loss of 3.91 per cent. The remaining 96.09 per cent. of the whole energy absorbed in each lamp, appears in the arc between its carbons.

The shunting or short-circuiting device in each lamp, consists of a small magnet core, surrounded by a coarse and a fine wire helix similar to those of the working magnet. No current passes through the coarse wire until the magnet which it surrounds has raised its armature. The latter, together with the coarse wire, then form a part of the short circuit established through the lamp. The fine-wire helix of the shunt is put in the circuit of the fine wire regulating helices before described. During the normal operation of the lamp, this fine-wire helix exercises a magnetizing influence on its inclosed core, which thus attracts its armature with a certain degree of force, but not enough to lift it. But when, through the exhaustion of the carbons in the lamp, or from their failing properly to feed together, the arc between them becomes considerably lengthened, thus developing an abnormally great resistance in the lamp, a greatly increased current will be shunted through the fine-wire helix, the iron core of the latter will raise its armature, and thus establish a circuit of low. resistance through the lamp, independent of its carbons.

Evidently, when the short circuit is closed as above, very little current circulates in the fine-wire helix, and the magnet would drop its armature and thus open the circuit, were the armature not retained by some means other than the magnetism due to this helix; but it will be remembered that the coarse-wire helix surrounding the same iron core is now brought into action, and by this means the armature is retained.

This simple cut-out mechanism effec tually guards against all the dangers of general extinction, which are always present when many lamps are burned in a single circuit. It will be noticed that it operates without any change in the strength of the general current operating a system of many lamps; and herein lies its great merit. No effect is produced in any lamp other than the faulty one, except a slight case of brilliancy after the "cut out" of the latter has acted. The extinction of a lamp necessarily calls attention to it at once, and the renewal of its carbons may be effected in a few seconds. As soon as the renewed carbons come into contact, a considerable portion of the current through the lamp is diverted from the magnet of the cut-out device, by the passage afforded through the carbons; thus the retaining magnet is so weakened that it drops its armature and restores the whole current to the carbons, and the light is re-established.

The carbons employed in these lamps are covered with a thin coating of copper, and are twelve inches long. They burn, without renewal, about eight hours, during which time about nine and a-half inches of the positive, and four inches of the negative are consumed.

When it becomes desirable to operate the lamps more than eight hours continuously, "double-rod" lamps are used. In each of these lamps two movable rods, and two sets of carbons are employed. The rods are placed three inches apart, and each is moved and controlled in the manner already described. But both rods are actuated by

a single magnet, the same as that employed in the single-rod lamp. The simple lifting mechanism connected with the magnet, is so arranged that one of the rods is lifted slightly in advance of the other. Now, when the electric current first passes through such a lamp, the two sets of carbons, having their members in contact, will divide the current between them; but as soon as the members of one set are separated by the action of the magnet, the whole current is thrown through the other set without showing any spark between the members of the set first separated. When the continued action of the magnet separates the remaining pair of carbons, the voltaic arc appears, and the light is established. It must now be evident that the clamp which was the last to raise its rod, will be the first to release it when a forward movement of the carbon becomes necessary. Hence, the set of carbons which first commenced to burn, will continue to do so until consumed; the other set remaining separated as at first. But when the burning carbons are exhausted, and can no longer move forward, any further effort of the magnet to feed them, will at once bring the reserve set of carbons into contact; the whole current will then pass through this set, leaving the other carbons without current, and permanently separated. The reserve set of carbons will now be separated by the magnet, and burn continuously. In practice, the transfer of the voltaic arc from one set of carbons to the other, is accomplished instantaneously, and is scarcely noticeable.

It will be seen that this simple arrangement cannot possibly fail in its function, there being no switch to get out of order, or contact surfaces to become burned; further, that it is perfectly automatic, and operates at the instant a change becomes necessary, and not sooner.

By means of these double-rod lamps, a system of lights may be maintained in continuous operation sixteen hours without requiring any attention. This is sufficient for the longest winter night. But by introducing three rods, and three sets of carbons in each lamp, the lights may be maintained quite as easily and certainly for twenty-four hours. In this case the clamps lift their rods successively, and feed them in the reverse order, as before.

A notable feature of the Brush lamps is the absence of any adjustment requiring the attention of the user; he has but to insert the carbons, and the lamps are always ready for action.

RESULTS ATTAINED.

On August 22d, 23d and 25th, 1879, a system of very careful measurements, both electrical and dynamic, were made by thoroughly competent persons for the purpose of accurately determining the efficiency of the Brush machine and lamps. These measurements were made at the works of the Telegraph Supply Co., Cleveland. The machine used was of the size known as No. 7; its individual mark, distinguishing it from other machines of the same class, being "Y." This machine differed in no respect from others of the same size, and there is no reason to suppose that its performance was above or below the average. Sixteen lamps of the usual pattern were employed, with about two hundred feet of copper line wire No. 10.

The machine was driven by a "Buckeye" engine with automatic cut off, running at a speed of 157 revolutions per minute. This gave to the machine an average speed, during all of the experiments, of 770 revolutions per minute. The normal speed of these machines is 750 revolutions per minute, at which speed they operate sixteen lamps normally, giving to each an arc of about two millimeters length. At the speed at which these measurements were made, the sixteen arcs were perceptibly lengthened, and, of course, more than the normal amount of driving power was required. But as this produces a nearly corresponding increase of current, no attempt was made to bring the speed nearer to the normal point, because of the difficulty of changing the speed of the engine.

The machine being entirely new, the bearings heated considerably. Some driving power must have been wasted on this account.

The first electrical measurements made, were for the purpose of determining the difference of potential existing at the terminals of each lamp. This was done by a method devised by the writer.

A battery of 48 small Daniell's cells was constructed on the "gravity" plan, carefully insulated, and freshly charged with sulphates of copper and zinc to ensure normal action. The sixteen lamps having been adjusted to furnish arcs as nearly equal as possible, the positive terminal of one lamp was connected with the positive terminal of the battery; while the negative terminal of the same lamp was connected with the negative end of the battery; a very sensitive galvanometer being interposed. Now, it is evident that if the difference of potential between the ends of the battery is greater than that between the terminals of the lamp, current will circulate in its normal direction through the battery, and will be indicated by the galvanometer; but if this potential is less than that of the lamp, current will also flow through the battery, but in a reverse direction, and will also be indicated by the galvanometer; while if the potential is the same in both, no current will pass in either direction through the battery, and the galvanometer will show no deflection.

As was expected, the potential of the battery proved to be higher than that of the lamp. By means of a simple arrangement, any number of the battery cells could be included in the circuit at pleasure. Such a number was chosen that the galvanometer indicated no current or currents fluctuating from zero, equally in both directions. The appearance of the arc in the lamp was then carefully noted, and the speed of the machine counted. From time to time the condition of the arc would change slightly, or the speed of the machine would vary a little, and then more or less battery would be necessary to effect the balance. When such changes occurred, another set of observations was made as before, and so on, until results corresponding to the average working of the lamp were secured. The large number of observations made, sufficiently eliminated the error due to the fact, that no fraction of a single cell of the battery could be used in the experiments. This method of measuring the difference of potential between the terminals of the lamp, proved to be extremely satisfactory and certain in its operation; the addition or subtraction of a single cell of battery being sufficient to deflect the galvanometer needle strongly to the right or left.

A series of observations was thus made on each of eleven of the sixteen lamps, selected at random, and as the results obtained from the various lamps agreed very closely, it was deemed unnecessary to carry the process further.

The difference of potential between the terminals of the *average* lamp was thus found to be equal to that of 42.46 cells of the battery, at an average speed in the machine of 770 revolutions per minute.

The next point to be determined was the resistance of the average lamp. For this purpose a resistance consisting of coils of coarse copper wire was substituted for one of the lamps, and made of such amount that 42 cells of the battery exactly balanced the difference of potential between the two ends of the resistance, while the speed of the machine was 770. This resistance wire weighed nearly seven hundred pounds, and was but slightly warmed during the experiment. Its resistance was then *immediately* measured, with much care, before any appreciable cooling could take place, and was found to be 4.51 ohms. From this resistance, balancing 42 cells of the battery, the resistance of the average lamp, corresponding to 42.46 cells of battery, is easily deduced, and is found to be 4.56 ohms. This, multiplied by sixteen, the number of lamps in circuit, gives the total resistance of the lamp circuit,—72.96 ohms. The resistance of the conducting wires between the machine and lamps was not measured, being so small as to be unimportant. The average total internal resistance of the machine, measured through the brushes, with the commutators in various positions, and all of the conductors warm from active use, was found by careful measurement to be 10.55 ohms. Variations of only about .02 ohm were observed when the commutator was turned in different positions. This resistance, added to that of the lamps, gives a total normal internal and external resistance of 83.51 ohms, 87.36 per cent. of which (72.96 ohms) is external. Hence, 87.36 per cent. of the current developed by this machine, is available for external work.

We have found that the electro-motive force of the current overcoming a resistance of 4.56 ohms (the resistance of one lamp), is equal to that of 42.46 cells of battery; hence the total electro-motive force of the current, overcoming the total resistance, is

$$\frac{42.46 \times 83.51}{4.56} = 777.59 \text{ cells.}$$

Assuming the electro-motive force of each cell of battery to be 1.079 volts, the electro-motive force is, $777.59 \times 1.079 = 839.02$ volts. By Ohm's well-known formula, the current circulation is

$$\frac{839.02}{83.51} = 10.04 \text{ webers.}$$

We have next to determine what portion of the energy of the whole current was utilized in the sixteen voltaic arcs, for the development of heat and light. To this end, the resistance of the sixteen lamps, including all connections, magnet wires, carbons, etc., minus the resistance of the arcs, was determined in the manner already described, and found to be 2.1 ohms. This, subtracted from the total resistance of the lamps, 72.96 ohms, leaves 70.86 ohms as the resistance of the sixteen arcs. This is 84.85 per cent. of the resistance of the entire circuit; and as the work performed by the current in any part of the circuit is directly as the resistance of the said part, it would appear that 84.85 per cent. of the entire energy of the current was expended in the arcs. But this is not the case, because one per cent. of the current passing through each lamp is diverted from the arc by the fine-wire adjusting helices as before explained. Deducting one per cent. of 84.85, we have left 84., expressing the percentage of the entire energy of the current, appearing as heat and light in the arcs.

The electrical measurements, above described, were made in the presence, and with the assistance of Mr. G. H. Wadsworth, Chief Operator and Electrician of the Western Union Telegraph Co.'s Cleveland Office.

The instruments used in measuring resistances, were a tangent galvanometer by Phelps, and a set of standard resistance coils rating from .01 ohm upward. The measurements were made by substitution.

During the progress of the electrical measurements above described, a system of careful measurements of the driving power absorbed by the machine was also made. This was done by Mr. Isaac V. Holmes, of Cleveland, a gentleman widely known as a mechanical engineer and expert. His measurements were made from indicator diagrams taken at the cylinder of the engine, and covered a period of about three hours.

The following table embodies the results obtained by Mr. Holmes:

Total power developed with the 16 light machine at 770 revolutions "closed"	18.73 H.P.
Less friction load of engine "light"	2.44 "
Less friction due to increase of load (18.73—2.44=16.29) at 5 per cent.	.81 "
Total power absorbed by 16 light machine at 770 revolutions "closed"	15.48 "
Total power developed with the 16 light machine at 770 revolutions "open"	4.23 H.P.
Less friction load of engine "light"	2.44 "
Less friction due to increase of load (4.23—2.44=1.79) at 5 per cent.	.09 "
Total power absorbed by 16 light machine at 770 revolutions "open"	1.70 "
Total power absorbed by 16 light machine at 770 revolutions, in the production of current (15.48-1.70)	13.78 H.P.

NOTE.—The terms "closed" and "open" refer to the external circuit of the machine. Thus, when the machine was "closed," it's current was working through the normal external resistance—that of the sixteen lamps. When "open," no current was generated.

These results agree very well with those obtained by other engineers with other machines of the same size and style. On February 6th, 1879, Mr. Noah R. Harlow, of the Lowell Water Power Co., measured the power absorbed by one of these sixteen light machines located in the Merrimack Print Works, Lowell, Mass. The speed of the machine averaged about 747 revolutions per minute while operating sixteen normal lights, and the total power absorbed under these circumstances was found to be 13.86 horse-power, including friction of dynamometer employed.

On March 18th, 1879, another machine was measured at the Dry Goods Depot of John Wanamaker, Philadelphia, Pa., by Wm. Lee Church, of the Buckeye Engine Co., of New York. Mr. Church employed the method of indicator dia-

grams, and obtained as a result 13.5 horse-power for the total power absorbed by the machine when operating sixteen lights. The speed of the machine was not given in his report, but was presumably about 750 revolutions per minute, as he was aware that this was the speed at which the machine should be run.

The higher results obtained by Mr. Holmes may be attributed to the increased speed at which his machine was driven, and the abnormal friction at the journal bearings, due to their newness.

Referring to the electrical measurements already described, we find that we have a current of 10.04 webers, with a total resistance of 83.51 ohms. Now the value in foot pounds of any current, is $C^2R \times t.737335$; wherein C is the current in webers, R the total resistance of the circuit in ohms, t the time in seconds, and .737335 the equivalent in foot pounds of one weber per ohm per second. Hence, the value in foot pounds per minute, of the current from the sixteen light machine is: $10.04^2 \times 83.51 \times 60 \times .737335 = 372410.58$. This divided by 33,000 $= 11.285$, which is the energy of the current expressed in horse-power. Again,

$$\frac{11.285}{15.48} = .729;$$

hence, 72.9 per cent. of the total power applied at the pulley of the machine was converted into current. We have already found that 84 per cent. of the entire energy of the current appeared in the voltaic arcs. Multiplying 72.9 by .84 we have 61.24 as the percentage of the total driving power appearing in the arcs.

If we deduct friction and resistance of air from the gross power absorbed, and consider only the power actually absorbed in the production of current as is usually done in determining the efficiency of dynamo-electric machines, we have,

$$\frac{11.285}{13.78} = .8189;$$

or 81.89 per cent. of the absorbed power converted into current. As before, 84 per cent. of this current appearing in the arcs, we have $81.89 \times .84 = 68.79$ per cent. of the entire power absorbed in the production of current, present as heat and light in the sixteen arcs.

RECAPITULATION.

Resistance of dynamo-electric machine	10.55	ohms.
Resistance of external circuit	72.96	"
Total resistance of circuit	83.51	"
Resistance of 16 voltaic arcs	70.86	"
Percentage of current available for external work	87.36	"
Percentage of current appearing as heat and light in 16 voltaic arcs	84.00	"
Electro-motive force of current	839.02	volts.
Volume of current	10.04	webers
Total driving power required	15.48	H. P.
Driving power absorbed in production of current	13.78	"
Energy of current expressed in horse-power	11.285	"
Percentage of gross power converted into current	72.90	"
Percentage of absorbed power converted into current	81.89	"
Percentage of gross power appearing in arcs	61.24	"
Percentage of absorbed power appearing in arcs	68.79	"

These results require no comment. Their excellence is apparent. They have not yet been approached by those of any other dynamo-electric apparatus operating multiple lights; or even by those operating single lights.

Mr. Louis Schwendler, in his excellent "report on the results obtained by the *electric light experiments*," dated London, Nov. 1st., 1878, gives the percentage of absorbed power appearing in the electric lamp, in the cases of four dynamo-electric machines. The lowest result given was 30 and the highest 62 per cent.; the latter with a machine of the Siemens type. This is much the highest efficiency we have yet seen reported, authentically, of any dynamo-electric machine. The result was obtained only under the most favorable circumstances, as pointed out by Mr. Schwendler, viz: with a machine furnishing a single light, and developing a large volume of current (29.5 webers) of comparatively low electro-motive force.

The Brush machine, developing a much smaller volume of current, of a vastly greater electro-motive force, and operating sixteen lights, yet shows a much higher efficiency, viz.: 68.79 per cent.

From the peculiar nature of the Brush system of lighting, arise very important advantages over other systems, not yet touched upon. We refer to the great ease with which the lights may be oper-

ated a long distance from the dynamo-electric machine, and the small loss of effect occasioned by so doing.

When a separate dynamo-electric machine is employed for each light, accordto the systems which appear to be most in favor in England and France, or when the same machine furnishes several currents, each operating a light, then each lamp must be independently connected with the machine, and the whole loss of current, occasioned by the resistance of the pair of conducting wires, must fall upon each single lamp. There must also be as many complete sets of conductors as there are lamps.

In the Brush system, one pair of conductors is sufficient for all the lamps, no matter what their number may be; and the loss due to these conductors falls not on each lamp, but is equally divided between all. Hence, in order to reduce the loss in each light due to the conductors in the single-light system, to the same amount as that in each light in the multiple-light system, the size of the conductors in the former case must be to that of the conductors in the latter case, directly as the number of lights operated in the multiple-light system, to unity. And, since the single-light system requires as many sets of conductors as there are lamps, while the multiple-light system requires but one set of conductors, the total weight of conductors required for a given number of lamps, (keeping the total loss of effect due to this cause the same in both systems,) is as one, for the multiple-light system, to the square of the number of lamps for the single-light system. To illustrate: suppose sixteen lights are operated at a given distance from the dynamo-electric machine or machines, and the loss of current due to resistance of conduct is limited to a certain amount; and let the weight of conductors required in the multiple-light system be represented by 1; then the weight of conductors required in the single-light system will be $16^2=256$.

Hence, in operating a large number of lights at a long distance from the source of power by the single-light system, either an enormously expensive system of conductors must be employed, or else a great part of the whole energy of the currents will be wasted in overcoming the resistance of the smaller conductors.

This objection to the single-light system is fatal, when any considerable separation of the machines and lamps becomes necessary.

The Brush system of lighting being free from this defect, the lights may be maintained at a great distance from the source of power with very little loss of effect, and by means of conductors of moderate size and cost. The resistance of number 10 (Stubb's gauge) copper wire of commercial purity, is about .6 ohm per thousand feet; therefore 7600 feet of such a conductor will equal in resistance, only one of the sixteen lamps already described.

$$\frac{4.56.}{.6}\times 1000=7600.$$

so that fifteen lamps of full power could be operated through such a conductor of this length; or the whole sixteen, with a loss of only about six per cent. in effect. Only fifty per cent. of the total effect would be sacrificed by working through such a conductor more than eleven miles in length, (60800 feet) and this loss might be reduced one-half by doubling the size of the conductor, which would still be of moderate dimensions.

No reference has yet been made to the use of more than sixteen lamps in the circuit of the No. 7 machine; but 17 or 18 lamps are often employed with the machine running at its normal speed of 750 revolutions per minute, and with good effect, although the total light produced is less than with sixteen lamps. At the speed at which the above described measurements were made, viz: 770 revolutions, 19 or 20 lamps may be burned with tolerably good effect, although the maximum amount of light is obtained with 16.

As many as 33 lamps have been operated simultaneously in the circuit of a No. 7 machine running at a speed of 800 revolutions per minute, with an arc of appreciable length in each lamp; but the total light produced was less than half that obtained when 16 or 17 lamps were employed. When the number of lamps operated by the No. 7 machine, running at full speed, is reduced below 16, the brilliancy of the remaining lights is of course increased, but not in proportion

to the reduction in number. Hence the total amount of light is diminished. The *quantity* and *intensity* of the current are such as to produce the maximum light in 16 arcs.

At the time Mr. Holmes measured the power required to drive the sixteen-light machine, he also measured that required by a six-light machine of the size and style known as "No. 5."

This machine furnished six beautiful lights in single circuit, similar to those of the No. 7 machine, but with somewhat shorter arcs. (It will be remembered that the No. 7 machine was driven above its normal rate of speed.) The total power required to drive this machine was 4.5 horse power. The following are Mr. Holmes' figures:

Total power developed with the 6 light machine at 922 revolutions "closed"	7.18	H.P.
Less friction load of engine "light"	2.44	"
Less friction due to increase of load (7.18—2.44=4.74) at 5 per cent.	.24	"
Total power required by six-light machine	4.50	"

No measurements of the current from this machine were made.

REMARKS.

Much has been written by eminent electricians, and others, about the enormous loss of total lighting effect which follows the use of more than one center of light in an electric circuit. Some state that the total light diminishes as the square of the number of lights increases; others, that the loss is as the cube of the number of lights.

These extraordinary assertions are made without any reference to the energy or character of the current employed, and are never supported by any experimental data.

The experience of the writer has uniformly shown that when a small amount of electrical energy is expended at several points, very little or no light results; while if the same energy be properly expended at one point, a large amount of light may be evolved. On the other hand when a *large* amount of current energy is properly expended at a number of points not too great, the total light produced may nearly equal that evolved when the whole energy is concentrated at one point.

A certain amount of energy is required before any light at all is produced, and if the division falls below this point nothing in the way of light results. But after a certain point is reached, at which a good light is produced, then the increase of light appears to be almost directly as the increase of energy exhibited in the arc, the length of the latter remaining constant. Mr. Schwendler's experience here appears to coincide with our own. In his report, before mentioned, he says: "If we make the highly probable supposition that the resistance of an arc of constant length is inversely proportional to the current which passes through, then the light produced would be proportional to the current. This appears to be the case." Again he says: "Although the light must be very nearly proportional to the total energy consumed in the arc, the resistance of the arc decreasing with the increase of current, it follows that the light *cannot* be proportional to the square of the current."

It is argued by many, that an increase of energy in the arc produces a corresponding increase of temperature of carbons, and, according to a well known law, the light must increase much faster than the temperature. But in fact, after a certain point is reached, the temperature of the positive or light-giving carbon does *not* increase with increase of current, but the area of surface heated to maximum, increases almost directly with the current. The "certain point," several times alluded to, is at that stage of process when vaporization takes place over a well defined spot on the positive carbon. Obviously, when rapid evaporation has commenced, the temperature of the carbon cannot be increased at that point, and an increase in the amount of heat evolved, can only result in an increase in the rapidity of vaporization, over an increased area.

A careful study of the voltaic arc, and of the carbon points after being used with currents of various strength, cannot fail to convince the most skeptical, of the truth of these assertions.

Quite contradictory to the spirit of those portions of Mr. Schwendler's report, already quoted, he says in another place: "If more than one light is produced in the same circuit, by the same current, the external or available light becomes rapidly dearer with increase of the number of lights produced. For this

reason already, if not for many others, the *division of light* must result in an engineering failure."

It seems scarcely possible that Mr. Schwendler could have overlooked the obvious fact, that a current representing a given amount of energy, and adapted to produce the maximum light from one center, may, by having its dimensions suitably altered, *i.e.*, its electro-motive force increased, and its volume diminished while its energy remains the same, 'be adapted to produce its maximum light from several small centers, instead of from one large one (provided, of course, the energy at each center is not reduced below the "certain point" before defined). Yet he seems to have overlooked, or entirely ignored this fact.

The performance of the No. 7 Brush machine with its sixteen lamps, already detailed, proves unquestionably that the division of a large current energy among many lights in the same circuit, or, more properly, the *multiplication* of lights on a single circuit, has resulted in a great engineering *success*.

The votaries of the single-light system would advocate in place of the single No. 7 Brush machine, the use of *sixteen* small machines, with their costly sytem of conductors. *Each* of these would require as much attendance, and involve as much expense for repairs, as the single large machine. At least fifty per cent. and probably one hundred per cent. more driving power would be required to produce sixteen lights of the same size, and the first cost of the apparatus would be vastly greater. Further, if the lights were required at any considerable distance from the source of power, the single-light system must prove *entirely* impracticable.

THE ACOUSTICAL UNIT OF DIMENSIONS OF ROOMS.

By Lieut. F. J. DAY, R.E.

From "Royal Engineer Institute."

Many physicists have written upon the science of acoustics, but few, if any, have considered the subject with a view to practically assisting the architect in the proportioning of halls and rooms, so as to make their contained air vibrate to any musical note that may be communicated to it.

The following paper is therefore submitted to the Corps in the hope of promoting a discussion on the important question of determining the proper unit of dimensions of rooms, in order to make their contained air vibrate to any note that may be communicated to it; thereby engendering those properties in the room which are technically summed up in "the room being good for sound,"—and, if possible, discovering a few simple rules for the guidance of architects in selecting a suitable unit of dimensions for their projected structures, whereby they may ensure the acoustical success of their designs.

The sounds imparted to the air of a building would be those of the human voice or of some musical instrument, of which we may take the organ as a type, for this instrument contains stops which represent the sound produced by every description of instrument, the sounds generated only differing from each other in color.

This brings us to the consideration of the component parts of the musical note produced by any instrument, viz:—

The pitch, intensity and color. The pitch of the musical note is regulated by the number of vibrations of the sound wave per second, the greater the number of vibrations the higher the pitch, and as sound travels with a uniform velocity, the greater the number of vibrations the shorter the length of the waves, therefore the shorter the length of the waves the higher the pitch of the note.

The intensity of the sound depends upon the amplitude of vibration, or, in other words, on the amount of disturbance generated, but has nothing to do with the length of the waves producing the individual note, nor with the nature of the disturbance.

The color or *timbre* of the note de-

pends upon the generating instrument; the notes are produced in different ways, and though the pitch be the same, they differ in quality or character according to the nature of the producing instrument. Thus if a particular note is sounded first on the piano, then on the organ, and successively on the flute and violin, or by the human voice, a great difference will be noticed in the sounds produced; this difference is due to many causes; for instance, in the piano the note is struck and the vibration produced, but the intensity rapidly decreases; in the organ the note is continued as long as the air is blown into the pipe; in the flute there is a rushing sound of the wind being blown into the instrument. But the chief reason for the difference is the production of overtones or harmonics of the fundamental note in different intensities by the various instruments; these harmonics not only differ for the various sounding bodies, but even for the same body when sounded in different ways.

We shall see hereafter that any body that will resound to the fundamental note will also resound to its harmonics, so that whatever arrangements are made for the air spaces to resound to the fundamental notes will also hold good for its harmonics. Although the production of the various sounds does not come within the province of the architect, yet he should understand their formation in order to arrange for their reception, and to be able to proportion the various air spaces so as to take up any note communicated to them. It will be well, therefore, to inquire cursorily into the method by which the various notes are produced.

As the stop on an organ representing any instrument differs from the instrument itself, only in the proportion of the harmonics generated in each case, it will simplify our investigation to take the organ as a general representative of musical instruments, and we shall show that if our air space is proportioned to any fundamental note it will also take up any harmonics of that note which may be communicated to it. We have, therefore, only to inquire into the production of sounds by the organ and the human voice.

As regards the organ, we need not trouble ourselves with the complicated mechanism by which the various results are attained; suffice it to say that the instrument consists of a case containing many series of pipes, into any of which the wind is admitted by the double action of the stops and keys. The opening of the stops enables the wind to pass into the particular channels with which the several pipes communicate, the openings of these channels being stopped by valves or pallets, which pallets are worked by the keys. On the key being depressed the pallet is opened, and a jet of wind is driven into the foot of the pipe, which sets the lip or reed into vibrations, and thus generates the note required.

All organ pipes are constructed either of metal or of wood, and are distinguished by the names of flue and reed pipes. The flue pipe, when made of metal, consists of a body and a foot; the body is generally a cylinder having a small portion towards its lower end slightly flattened inwardly, so as to produce a straight edge; the part thus pressed down does not extend to the bottom of the body of the pipe, but a small portion at its lower extremity is cut off, the edge thus formed being called the upper lip. The foot of the pipe is of conical form, and has a straight edge formed similarly to the one on the body of the pipe, and which is termed the under lip. The top of the foot is closed at its lower broad end by a circular metal plate called a "langward;" a segment of this is cut away so as to produce a straight edge parallel to the under lip, leaving a narrow fissure or flue between them directly underneath the straight edge of the upper lip. The body and foot are soldered together with the lips exactly opposite each other, and the aperture thus formed constitutes the mouth of the pipe.

Wooden flue pipes are of rectangular section, and are constructed on the same principle as metal pipes; they have upper and under lips, a langward, and a narrow fissure to discharge the jet of wind which is admitted into the foot of the pipe and which strikes against the upper lip.

Flue pipes are made to speak by sending a jet of wind into the foot of the pipe; it rushes through the narrow

fissure above described, and, by striking against the upper lip of the pipe, sets it in vibration. These vibrations are communicated to the air in the pipe, which is thus set in motion, and sounds the note which the pipe is constructed to produce; the length of the pipe must, of course, be proportioned according to the rate of vibration of the upper lip, which is regulated by the pitch of the note it is intended to sound.

Reed pipes are generally made of metal, the body of the pipe being either cylindrical or conical in form; a cylindrical block of metal with a flange round the upper end to prevent its sinking too deeply into the foot of the pipe, is fixed at its mouth; the reed is a small tube of which a portion is cut away lengthwise; it passes through the center of the block and is attached to it—the tongue is a thin piece of metal slightly bent, and attached to the reed in such a way as nearly to close up that portion of the reed which has been cut away; a tuning wire passes through the block, the lower portion of which is bent so as to press against the tongue, while its upper part is a little crooked, and is filed in a notch so as to receive the tuning knife; by this means it can either be raised or depressed, and the vibrating portion of the tongue thus lengthened or shortened to sharpen or flatten the pitch of the pipe.

The peculiar tone of the reed pipes arises from the fact of the wind, rushing through the opening between the tongue and the reed, causing the tongue to vibrate; the quicker the vibrations the more acute will be the pitch of the pipe; the length of the pipe is, of course, proportional to the pitch of the note produced by the reed.

From the above we find that all the notes of the organ are produced by setting either the lips or reeds of the pipes in vibration; these vibrating surfaces communicate their movement to the air in the pipes, which, being properly proportioned to the pitch of the note, resound to it, and the pipe is made to speak. From this we may gather that the pipe has nothing to do with the formation of the sound, but, only, owing to its length being proportional to the time of vibration of the generating medium, its contained air resounds to the note given out, which thus reinforced becomes audible.

We have next to consider the more difficult subject of the production of sounds and words by the human voice, and in order to do this we must discuss at some length the method of generation of musical notes, vowel and consonant sounds by the human voice.

We proceed first to examine the organs of the human frame which generate and articulate sound, and find at the top of the wind-pipe an apparatus which leaves for the passage of air only a long narrow slit in the back and front direction, which is called the "glottis." The sides of this slit are not solid masses of animal matter, but elastic bands or ligaments, which, though not very deep vertically, have the power of vibrating to the right and left. The ends of these ligaments are held in position and in a state of tension by different muscles (these, however, have nothing to do with this investigation, which only deals with the actual generation of sound). These elastic bands are called the vocal ligaments, and we must next consider their condition under various circumstances.

When the voice is in a state of rest the vocal ligaments are not stretched with any particular force, and the ends of the right and left ligaments are not pressed together, but, on the contrary, kept apart by certain muscles exerted only with this object. Under these conditions the aperture is sufficiently wide to allow the breath to pass freely, and the ligaments not being stretched there is no cause for setting them in vibration.

But when a sound is to be produced, other muscles come at once into play, the ends of the two ligaments are pressed together, although not probably closely, and, at the same time, the ligaments themselves are extended by other muscles to any required degree of tension.

If then air be forced from the lungs through the "glottis," it rushes with great rapidity through the small aperture, and impinges upon the sides of the ligaments, which are in a state of tension and ready to be set in vibration, thus producing a musical note. The pitch of the note generated, of course, depends upon the degree of tension of the ligaments.

We thus see that to utter a musical

sound two sets of muscles are put into action; first, in order to utter a musical sound the right and left ligaments are pressed together, and second, in order to produce a note of a particular pitch the ligaments are stretched to a particular tension, their vibrations being made of longer or shorter duration according as a note of high or low pitch is required. This action is imitated by the organ builder in every reed pipe he constructs; the closing of the ligaments is imitated by introducing the tongue into the portion of the reed which has been cut away, and thereby nearly closing the aperture, and the state of tension is produced by raising and lowering the crooked wire, which presses against the tongue, and thereby lengthens or shortens the vibrating surface and flattens or sharpens the pitch of the note.

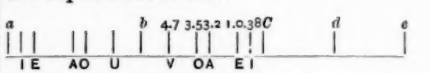
The cavity of the mouth performs to the vocal ligaments the same functions as the body of the organ pipe does to the reed, the air in the resonant cavity of the mouth reinforcing the sounds produced by the vocal ligaments, in the same way as the organ pipe reinforces the note produced by the reed.

Thus far we have traced a likeness between the method of generation of the note produced by the human voice and those of a reed pipe of an organ; but we must now inquire into the method of producing vowel sounds by the human voice, and for this purpose may adduce an experiment made by Professor Willis, and recorded in Volume III. of the *Cambridge Transactions*.

In the experiment he uses a simple pipe, which admits of its length being varied at pleasure. Air is blown through a long channel which terminates in a reed that gives a note of known pitch, and the length of whose air wave is therefore also known; the carrier of this reed is fitted accurately, but not tightly into the pipe, whose length can thus be made variable; the tube is now slid along so as to vary the position of the reed carrier, which thus becomes a plug in the pipe, by varying the position of which, different lengths of pipe are left between it and the external air. Professor Willis then points out the following facts:—

"First, that in order to perceive clearly a vowel sound, it is necessary to sound different vowels in succession, the principal effect being produced by contrast, and no distinct vowel sound being impressed on the ear when the aparatus is maintained steadily in the arrangement proper for producing any one vowel.

"Second, the fundamental result of the experiment is this—



"In the above figures let *a* denote the place where the waves of air enter immediately from the reed (the reed being supposed to be at the left hand and the current of air being blown from left to right), and suppose the tube of variable length to extend from left to right, its mouth sometimes stopping at I, sometimes being advanced to E, A, O, &c. Also let $ac = bd = ce$ length of the sound wave produced by the reed. Then, when the mouth of the pipe is at the point I, it utters the vowel sound "I" (in continental pronunciation) the same vowel sound as in the word "see." When the mouth of the pipe is at E the vowel sound is that of "E" sliding between the vowel sounds in "pet" and "pay." When the mouth of the pipe is at A, O, U, the sounds are respectively those of "paa" and "part," followed by "pair," "nought" in "no" and "but" followed by boot." As the mouth of the pipe is carried towards *b* (the point bisecting *ac*) sounds become indistinct, and vowel sound is lost; or the only sound perceptible is that of our short U. On approaching *c*, the same vowels re-occur, but in opposite order. On proceeding further still, the same phenomena recur after an addition to the "pipe length," of the "length of the air wave," double the "length of the air wave," &c., but all sounds become less forcible.

"Now, upon varying the pitch of the reed (that is upon varying the length of the sound wave or the length of the spaces *ab*, *bc*, *cd*, *de*,) the lengths *a* I, *a* E, *a* A, *a* O, *a* U, *c* E, &c., remain unaltered. And when a reed of high pitch is used, or when the spaces *ab*, *bc*, &c., are made very short, some of the vowels U, O, &c., are lost. This accords with the experience of singers of high pitch, who can sing no vowel but I (sounded as in "see"). The distances of the vowel po-

sitions from c or from e are as follows: the distance when measured from a is rather less than in the other cases:—

"c I = e I = 0.38 inches.
c E = e E = 0.6 " to 1.0 "
c A = e A = 1.8 " to 3.2 "
c A′ = e A′ = 3.05 " to 3.08 "
c O = e O = 4.7 "
c U = e U = indefinite."

From these phenomena Professor Willis draws the following deductions:—

That from the air of invariable pressure at the mouth of the pipe there is a kind of reflection of the sound wave inwards, and thus every reed wave traveling along a pipe is reflected from the open mouth at a time depending upon the length of the pipe, the relation of which time to the time of the next reed wave will be different for different lengths of the pipe, thus producing a mixed wave, whose quality varies with the changes of that relation. This amounts to nearly the same as saying—that each puff of air through the reed may create a wave which travels with organ wave velocity coexisting with the one which follows the laws of resonance.

The following experiment pointedly illustrates this law. If a quill be snapped by the teeth of a wheel in rapid rotation a musical note is produced; but if, instead of a quill, a highly elastic spring is used, itself competent to give a musical tone, then a vowel sound is produced, and the name of the vowel depends upon the relation between these two musical notes, which relation is altered by grasping the spring at different points.

From this we infer that the actual vowel sounds are produced by a secondary wave which is coexistent but independent of the pitch wave, although it conforms closely to it; and, therefore, if we proportion our building to resound to the pitch wave, it will also accomodate itself into the secondary which produces the vowel sound.

We have now only to inquire into the production of consonant sounds, about which there is little to remark. They depend on the mode of beginning or ending an utterance of a vowel sound. Sometimes this is done at the glottis, sometimes at the beginning of the utterance by lowering the tongue from the palate, sometimes by opening the lips, sometimes by opening the teeth. In some cases a vowel sound must be formed before opening the lips, thus a momentary dull vowel sound within the mouth before opening the lips appears necessary to give the sound "bee;" if there be no such antecedent dull vowel sound, the sound emitted will be "pee." In closing the utterance of the vowel sounds, there is nearly the same variety. All these different modifications give rise to different consonants, but they do not appear to involve any particular principle which requires our attention. The rolling sound of the "r," the hissing sound of "s," and the guttural sounds of "och" and "ach," which seem to be produced rather in the palate than in the throat, also the sounds of "sh" and "th," appear to be abandonments of musical utterances, and probably do not require any action of the vocal ligaments, their peculiarities being given by the tongue, teeth, cheeks, and lips.

We find, therefore, that the vibrations of sound produced by the consonants are caused by cutting off either the beginning or ending of the vowel sound in all cases before it leaves the lips of the speaker, and, therefore, for our purpose we may say, before it has left the source of generation. Further, we have found that a column of air will resound to a note, giving a vowel sound, and it needs no demonstration to show that it will resound to any portion of this sound, or, in other words, to any variation produced by the addition of consonants. And in whatever way this vowel sound has been modified in its production, it will be similarly modified in its reproduction, for there is no external cause, which, by acting upon it, could make any alteration. We conclude that if air contained in a building will resound to the pitch wave of a note it will also resound to the wave when modified, either by a vowel or consonant sound, or their combination into words.

For the purpose of architectural construction we have, therefore, reduced all sound waves, produced by either the human voice or musical instruments, to the dimensions of those of the pipes of the organ of known lengths, and are

now in a position to discuss the most suitable unit of dimension for rooms.

The first point to be considered is whether the laws which govern the undulations of columns of air in organ pipes will hold good for columns of air in rooms?

This question cannot be better answered than by Professor Airy's description of a Plane Wave of three dimensions (*Airy's Sound and Atmospheric Vibrations*, p. 38).

"If we have a great number of pipes side by side with waves of a similar character passing simultaneously through all, so that the collateral condensations and pressures of air in the adjacent pipes will be the same, there will be no tendency of the air in one pipe to press sideways into another pipe; we may therefore remove the material boundaries of these pipes, and then we have air, extended in three dimensions, through which passes a wave, whose front is a plane, that is, in which all the points of similar motion and similar density are always in one plane."

Suppose, therefore, our room to be filled with organ pipes all radiating from the mouth of the speaker, the same wave passes through each pipe, and the result will not be affected if we remove the material boundaries of the pipes; let these be removed and we have a sphere of sound obeying the same laws as the column of air contained in the organ pipes.

The architect, of course, has nothing to do with the generation of sounds, but has only so to proportion his building that its contained air will take up and intensify any sound that is communicated to it. The method of generating the various sounds produced on the organ—to represent the human voice and musical instruments—is too long to give in this paper, and we therefore take it for granted that different lengths of pipe will resound to any note of any instrument that may be communicated to them, and for the details of their construction must refer the reader to Chapter XXIII. of Hopkins' *History of the Organ*.

The problem before us is to make the contained air space of a building take up any musical note that may be communicated to it. We propose to solve the problem by finding, first, a length which will resound to all the eight notes of the musical octave, and then to combine this length with those of the note "C" on the human voice and the various stops of a large organ.

In Ganot's *Physics* (Art. 207, p. 163, Ed. 1866) we have: "Hence if m denotes the number of double vibrations corresponding to the note C, the number of vibrations corresponding to the remaining notes will be given by the following table:—

"C	D	E	F	G	A	B	c
m	$\frac{9}{8}m$	$\frac{5}{4}m$	$\frac{4}{3}m$	$\frac{3}{2}m$	$\frac{5}{3}m$	$\frac{15}{8}m$	$2m$"

To produce which alteration in the number of vibrations the difference in the length of the pipes or waves must be:

$$D=\tfrac{8}{9}C,\ E=\tfrac{4}{5}C,\ F=\tfrac{3}{4}C,\ G=\tfrac{2}{3}C,$$
$$A=\tfrac{3}{5}C,\ B=\tfrac{8}{15}C,\ c=\tfrac{1}{2}C.$$

we have therefore to find a length which will be a multiple of all these lengths, or, in other words, to find the least common multiple of the fractions

$$\tfrac{8}{9},\ \tfrac{4}{5},\ \tfrac{3}{4},\ \tfrac{2}{3},\ \tfrac{3}{5},\ \tfrac{8}{15},\ \tfrac{1}{2},$$

or reducing to the least common denominator of

$$\frac{160, 144, 135, 120, 108, 96, 90}{180}$$

This we find to be $\frac{4320}{180}$ or 24.

The number 24 is a multiple of all the fractions representing the difference in length of the waves between that sounding C and its octave, and if the length sounding C be denoted by l, $24l$ will be the least length which will be a multiple of the lengths of the waves sounding the octave.

We next have to combine this with the various stops of a large organ; to do this let us take the specification of an organ with 87 stops given in Hopkins' *History of the Organ*, page 326:—

GREAT ORGAN, 22 STOPS.

1.	Sub Bourdon to tenor c key	32 ft. tone.
2.	Double open Diapason	16 "
3.	Bourdon	16 "
4.	Open Diapason	8 "
5.	Open Diapason	8 "
6.	Gamba	8 "
7.	Stopped Diapason	8 "
8.	Clarabella to tenor c	8 "
9.	Quint	$5\frac{1}{3}$ "
10.	Principal	4 "
11.	Principal	4 "
12.	Flute	4 "
13.	Twelfth	$2\frac{2}{3}$ "

14. Fifteenth.... .... 2 ft. tone.
15. Piccolo ..... 2 "
16. Full Mixture, III. Ranks..... 2 "
17. Sharp Mixture, V. Ranks..... 2 "
18. Cornet, II., III., and IV. Ranks "
19. Double Trumpet .....16 "
20. Posaune ..... 8 "
21. Trumpet..... 8 "
22. Clarion..... 4 "

SWELL ORGAN, 20 STOPS.

23. Bourdon..... .....16 ft. tone.
24. Open Diapason..... 8 "
25. Gamba..... 8 "
26. Echo Dulciana..... 8 "
27. Rohr Gedact..... 8 "
28. Voix Celeste..... 8 "
29. Principal..... 4 "
30. Gambette..... ..... 4 "
31. Flute..... 4 "
32. Twelfth ..... $2\frac{2}{3}$ "
33. Fifteenth .... 2 "
34. Octave Flute..... 2 "
35. Mixture, V. Ranks..... 2 "
36. Echo Dulciana, Cornet V. Ranks 4 "
37. Double Bassoon.....16 "
38. Hautboy..... 8 "
39. Trumpet..... 8 "
40. Horn ..... 8 "
41. Clarion..... .. .... 4 "
42. Vox Humana..... 8 "

CHOIR ORGAN, 15 STOPS.

43. Lieblich Bourdon.....16 ft. tone.
44. Open Diapason..... 8 "
45. Lieblich Gedact..... 8 "
46. Flauto Traverso..... 8 "
47. Dulciana ..... 8 "
48. Keraulophon..... 8 "
49. Spitzflote..... 4 "
50. Dulcet..... .... 4 "
51. Flute..... 4 "
52. Twelfth ..... $2\frac{2}{3}$ "
53. Gemshorn..... 2 "
54. Flageolet..... ..... 2 "
55. Mixture, IV. Ranks..... $1\frac{1}{3}$ "
56. Corno di Bassetto ..... 8 "
57. Bassoon throughout..... ... 8 "

SOLO ORGAN, 12 STOPS.

58. Bourdon.....16 ft. tone.
59. Violin Diapason..... ..... 8 "
60. Flute Harmonic..... 8 "
61. Violino..... 4 "
62. Flute Octaviant..... 4 "
63. Piccolo Harmonic..... 2 "
64. Contra Fagotto.....16 "
65. Clarionet ..... 8 "
66. Hautboy ..... 8 "
67. Hautboy Clarion..... .... 4 "
68. Tuba ..... 8 "
69. Tuba Clarion..... 4 "

PEDAL ORGAN, 18 STOPS.

70. Double Open Bass, wood.....32 ft. tone.
71. Double Open Bass.....32 "
72. Open Bass, wood.....16 "
73. Great Bass, wood.....16 "
74. Violone..... .....16 "
75. Stopped Bass.....16 "
76. Great Quint Bass.....$10\frac{2}{3}$ ft. tone.
77. Principal Bass, Metal..... 8 "
78. Violoncello, wood..... . 8 "
79. Flute Bass..... 8 "
80. Twelfth Bass..... $5\frac{1}{3}$ "
81. Fifteenth Bass.. ..... 4 "
82. Mixture, IV. Ranks..... $3\frac{1}{3}$ "
83. Contra Posaune.....32 "
84. Posaune.....16 "
85. Bassoon.....16 "
86. Trumpet..... 8 "
87. Clarion..... 4 "

From inspection it will be seen that the least common multiple of the lengths of all these pipes or waves sounding C is 32 feet, and that of the Vox Humana stop is 8 feet; if, therefore, we give these values to l and combine them with the least common multiple of the musical octave, we shall obtain the lengths required, which will be as follows:

For any stop on the organ,
i.e., any instrument... $32 \times 24 = 768$ feet.
For the human voice... $8 \times 24 = 192$ feet.

These lengths are multiples of the lengths of any wave that may be sounded, but it may be doubted whether the lengthened column of air will resound to any particular note, the length of whose wave is only a fractional portion of the whole.

The following extract from Weale's *Acoustics*, page 10, seems, however, to set this point at rest.

"If the agitation in the pipe be made more intense, a point will be reached in which the air will divide itself into two equal lengths—and in each half of the pipe vibrations will go on precisely as would occur in a pipe of half the length. As these pulses have only half as far to go, but travel at the same unalterable speed, two of them will be heard where one was before. The next note that can be produced by more vigorous agitation is that following the division of the pipe into three lengths." This is corroborated by Hopkins in his *History of the Organ*, page 212, para. 885, where he states that a pipe will occasionally sound its octave when it is too much winded, and from other causes, and we even find that pipes are occasionally made to sound their harmonics purposely, as in the case of the Harmonic Flute (Hopkins, page 137) which is a flute stop of 8 feet pitch, but of which the pipes are of double length,

either blown by a heavy wind or simply copiously winded.

We may, therefore, infer that a given pipe will sound several notes provided they are fractional parts of the whole length, and our unit of length being a multiple of all the waves that can be communicated to it either by the organ, or by the human voice, it will take up any note sounded in the building.

These lengths, as they stand, are, however, practically useless, as the limits to which a speaker can be heard are about 92 feet to his front and 31 to his rear. The length must therefore be broken up in some way, and that this breaking or doubling is practicable may be inferred from Ganot's *Physics*, page 175, Ed. 1866, which runs as follows:—"It must be added that if a closed and an open pipe yield the same primary tone, the closed pipe must be half the length of the open pipe." The same fact is given in different words in Hopkins' *History of the Organ*, page 110, para 434, and moreover, we find that brass instruments which, when extended, would be of unmanageable length, are coiled up into a number of turns to make them portable. This shows that the sound wave may be doubled up without being affected, and we are, therefore, led to suppose that any division will answer our purpose, but were we to act upon this conclusion the result would probably be—that our room would reproduce every note communicated to it in the form of a noise and not in that of a musical note, and as this is a point which should be carefully guarded against, we describe the difference between a sound producing a musical note and a noise. The following is taken from the *Encyclopædia Britanica*, vol, i., page 107, Art. 104:—

"Besides the three qualities above mentioned (amplitude, timbre and pitch) there exists another point in which sounds can be distinguished from each other, and which, though perhaps reducible to difference of timbre, requires some special remark, viz: that by which sounds are characterized either as *noises* or as *musical notes*. A musical note is the result of regular periodic vibrations of the air particles acting on the ear, and therefore also of the body whence they proceed, each particle passing through the same phase at stated intervals of time. On the other hand, the motion to which noise is due is irregular and flitting, alternately fast and slow, and creating in the mind a bewildering and confusing effect of a more or less unpleasant character."

The object we wish to attain, therefore, is to divide the sound wave in such a manner that it may always be reflected evenly and regularly from the surfaces with which it comes in contact. For this purpose we must examine the way in which the air vibrates in pipes, and draw our conclusions by considering the air in the room to vibrate similarly to the air in a pipe closed at both ends. This cannot be better described than by Bernouilli's Theory, of which the following is a summary:

"In dealing with the theory of pipes we must treat the air precisely in the same manner as we have dealt with elastic rods vibrating lengthwise, a pipe stopped at both ends being equivalent to a rod fixed at both ends, a pipe open at both ends to a rod free at both ends, and a pipe stopped at one end and open at the other to a rod fixed at one end and free at the other. When, therefore, air within a pipe is everywhere displaced along the length of the pipe, two waves travel thence in opposite directions, and being reflected at the extremities of the pipe, there results a stationary wave with one or more fixed model sections, on one side of which the air is, at any moment, being displaced, in one direction, while on the other side it is being displaced in the opposite. Hence when the air on both sides of the node is moving towards it, there is condensation going on at the node, followed by rarefaction on the reversal of the motion of the air. As a stopped end prevents any motion of the air, a nodal section is always found there, and as at the open end, we may conceive the internal air to be maintained at the same density as the external air, we may assume that such end coincides with the middle of a vential segment."

From these assumptions, which form the basis of Bernouilli's theory of pipes, we infer—

That in a pipe stopped at both ends, as in a rod fixed at both ends, there must be a note at both ends, and that any overtones compatible with this

arrangement may be produced. From this it is evident that any harmonic division would answer our purpose.

These harmonic divisions are $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$, &c., but no fraction which has not unity for its numerator will do.

Here, however, we may fall into another error andgive the sounds, when reproduced by the building, a nasal tone, unless we exclude the uneven divisions from our list; for if the predominant units of the building were one of the uneven harmonic divisions, the result on the reproduction of the sound might be the same as that of a narrow stopped organ pipe which gives out a poor and even nasal sound, as may be gathered from the following extract from the *Encyclopædia Britanica* (vol. i, page 108):

"The same character of softness belongs also to these instruments in which the powerful harmonics are limited to the vibrations 2, 3, . . . 6: because the mutual interference of the fundamentals and their harmonics gives rise to concords only. The piano, the open organ pipe, the violin, and the softer tones of the human voice, are of this class. But if the odd harmonics alone are present, as in a narrow stopped organ pipe, and in the clarionet, then the sound is poor and even nasal; and if the higher harmonics beyond the 6th and 7th are very marked, then the result is very harsh (as in reed pipes)."

From this we infer that there is danger in admitting the odd harmonic divisions into our calculations, as by their introduction the ruling note might become one of the odd harmonics, and the sounds communicated to the air of the building be thereby distorted in reproduction. We therefore propose only to use the even divisions such as the $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$, &c. This will reduce the figures previously found as follows:—

(*See Table on following column.*)

Thus bringing the lengths first found down to practical dimensions.

Having found our units, we proceed to design our building; the object in view is to make the contained column of air a harmonic of any undulation that can be communicated to it. But the whole of the lengths found above are harmonics of any of the notes produced either by the human voice or a musical instrument, and it needs no demonstration to show

	For the Organ or any Instrument.	For the Human Voice.
Unity.	768 feet.	192 feet.
$\frac{1}{2}$	384 "	96 "
$\frac{1}{4}$	192 "	48 "
$\frac{1}{8}$	96 "	24 "
$\frac{1}{16}$	48 "	12 "
$\frac{1}{32}$	24 "	6 "
$\frac{1}{64}$	12 "	3 "
$\frac{1}{128}$	6 "	$1\frac{1}{2}$ "
$\frac{1}{256}$	3 "	9 inches.
$\frac{1}{512}$	$1\frac{1}{2}$ "	$4\frac{1}{2}$ "
&c.	&c.	&c.

that their multiples will either be harmonics or multiples of the same notes, therefore any lengths we take that are multiples of those above found will answer our purpose.

Having first determined the position of the speaker or the orchestra, and adopted a convenient unit, the height is made a certain multiple of this unit, the breadth a certain multiple, and the length before and behind the speaker a certain multiple, the best results being probably produced by giving these multiples some harmonic relation to each other. Here we find the advantage of having excluded the uneven harmonics from our previous calculations, in the fact that we are not forced to make each dimension an even multiple of the others, but may make one or more of them uneven multiples without danger of generating a predominance of the uneven harmonics, and thereby impoverishing the sounding properties of the columns of air contained in the buildings.

In conclusion we give the dimensions of a few well known rooms, viz.: the Free Trade Hall at Manchester has a height of 58 feet, a breadth of 78 feet, and a length of 123 feet to the front of the orchestra, and 135 feet to the back, which gives a unit of 3 feet, the height being 19 units, the breadth 26 units, and the length 41 or 45 units, being a relation between height and breadth of nearly two to three, and between height and length of nearly two to five. The Theatre of the Royal Institution has a length of $44\frac{1}{2}$ feet, a breadth of $59\frac{1}{2}$ feet, and a height of 29 feet, which practically gives a unit of three feet, the length being 15 units, the height 10 units, and the breadth 20 units, giving a relation between length and height of three to two

and between height and breadth of one to two. The new room constructed by Professor Donaldson, at Edinburgh, has a height of 48 feet, a width of 36 feet, and a length of 90 feet, giving a unit of six feet, the height being eight units, the breadth six units, and the length 15 units. The relation between breadth and height being three to four, and between breadth and length three to $7\frac{1}{2}$.

Here we have three rooms known as acoustical success, all of which embody the rules enumerated in the foregoing pages, and would therefore seem to point to the correctness of the theories set forth. The subject, however, is one that has seldom been considered, and errors may have crept in which can only be eradicated by free discussion. This paper is therefore submitted to the Corps for the purpose of eliciting comments from officers who have studied the subject, and with the hope of—in the end—obtaining definite rules for the guidance of architects in proportioning the internal dimensions of their structural designs, so as to ensure acoustical success.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION. PART II.

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IV.

§ 4.

CONJUGATE FUNCTIONS.

A very important and interesting class of fluid motions are those which occur in the flowing of the fluid through an orifice. The differential equation of continuity becomes simplified in this case, when the motion takes place in two dimensions only, the equation becoming

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}=o$$

If we introduce the complex-quantity $x+iy$ we shall find that this equation is satisfied by either the real or imaginary part of a function of this complex-quantity. We will write for convenience

$$z=x+iy$$

then x,y denoting the rectangular co-ordinates of a point in the plane of x, y; we can say that this point is given by z. Any function of z, as $f(z)$, we may represent, by separation of its real and imaginary parts, as

$$f(z)=\mathrm{U}+i\mathrm{V}$$

For example,

$$f=\frac{(x+iy)^2}{2+3i}$$

this becomes

$$\frac{(2-3i)(x^2-y^2+2ixy)}{4+9}$$

and again

$$\mathrm{U}=\frac{2(x^2-y^2)+6xy}{13},\quad \mathrm{V}=\frac{4xy-3(x^2-y^2)}{13}$$

In this case we see that if we equate U and V each to a constant, that these equations will be the equations of two circles, and moreover, that these circles are orthogonal, since we have

$$\frac{d\mathrm{U}}{dx}\frac{d\mathrm{V}}{dx}+\frac{a\mathrm{U}}{dy}\frac{d\mathrm{V}}{dy}=o$$

Again let

$$f(z)=\sin z=\sin(x+iy)$$

or

$$f(z)=\sin x\frac{y\ -y}{\varepsilon+\varepsilon}{}_{2}+i\cos x\left(\frac{y\ -y}{\varepsilon-\varepsilon}{}_{2}\right)$$

from which

$$\mathrm{U}=\sin x\frac{y\ -y}{\varepsilon+\varepsilon}{}_{2},\ \mathrm{V}=\cos x\frac{y\ -y}{\varepsilon-\varepsilon}{}_{2}$$

If we make y constant and equal a, these will give us by elimination of x,

$$\left\{\frac{\mathrm{U}^2}{\frac{a\ -a}{\varepsilon+\varepsilon}{}_{2}}\right\}^2+\left\{\frac{\mathrm{V}^2}{\frac{a\ -a}{\varepsilon-\varepsilon}{}_{2}}\right\}^2=1$$

And similarly by making x constant and $=b$, we find

$$\frac{U^2}{\sin^2 b} - \frac{V}{\cos^2 b} = 1$$

the equation of a hyperbola. The ellipse corresponding to a line drawn parallel to x, is orthogonal to the hyperbola corresponding to a line drawn parallel to y. We have by differentiation of f

$$\frac{df}{dx} = f'(x+iy)$$

$$\frac{df}{dy} = if'(x+iy)$$

and again

$$\frac{d^2f}{dx^2} = f''(x+iy)$$

$$\frac{d^2f}{dy^2} = -f''(x+iy)$$

from which follow

$$\frac{df}{dx} + i\frac{df}{dy} = o$$

$$\frac{d^2f}{dx^2} + \frac{d^2f}{dy^2} = o$$

In the first of these writing for f its value of $U+iV$, we have, on separating real from imaginary,

$$\frac{dU}{dx} = \frac{dV}{dy}$$

$$\frac{dU}{dy} = -\frac{dV}{dx}$$

and the second becomes

$$\frac{d^2(U+iV)}{dx^2} + \frac{d^2(U+iV)}{dy^2} = o$$

giving also

$$\frac{d^2U}{dx^2} + \frac{d^2U}{dy^2} = o$$

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} = o$$

We have also from the expressions for the first derivatives

$$\frac{dU}{dx}\cdot\frac{dV}{dx} + \frac{dU}{dy}\frac{dV}{dy} = o$$

or in general the loci represented by the equations

$$U = \text{const. and } V = \text{const.}$$

are orthogonal. So we can in our case of fluid motion evidently let one of these equations U=const. represent the equipotential surfaces in the fluid, then manifestly the equation V=const. is the equation of the lines of flow. If V=const. be the equation of a surface, it will evidently be the bounding surface of the fluid, that through which the flow$=o$. Suppose that we have a certain region Σ in the plane xy, throughout which the function $f(x+iy)$ is single valued and continuous. The integral

$$\int f(z)\,dz = \int f(x+iy)\,d(x+iy)$$

taken in a positive direction around the boundary of this region will be$=o$. Represent this integral by J, we have then

$$J = \int (U+iV)\,d(x+iy)$$

or what is the same thing

$$J = \int (U\,dx - V\,dy) + i\int (V\,dx + U\,dy)$$

Now by referring to the relations

$$\frac{dU}{dy} = -\frac{dV}{dx}, \quad \frac{dV}{dy} = \frac{dU}{dx}$$

it follows that the quantities

$$U\,dx - V\,dy, \quad V\,dx + U\,dy$$

are exact differentials, and since f is continuous and single valued throughout the given space, it follows easily that

$$J = o.$$

Suppose now that the function f does not remain continuous and single valued throughout the region ε. Suppose this region to be enlarged by an amount $abcd$ as shown in the figure.

Call the new region Σ' and the corresponding line integral J′, and further assume that in the region $abcd$ the function f is single valued and continuous. The integrals J′ and J taken in a positive direction around the bounding lines of the regions Σ' and Σ will give rise to the equation

$$J' - J = [adc] - [abc]$$

where the symbols $[adc]$ and $[abc]$ represent the line integrals of

$$\int f(z)dz$$

over the curves adc and abc. Now

$$[abc]=-[cba]$$

and consequently

$$J'-J=[adc]+[cba]$$

The right hand side is evidently the integral

$$\int f(z)dz$$

over the boundary of the region $\Sigma'-\Sigma$ and consequently by the foregoing theorem is$=o$ *i.e.*,

$$J'=J.$$

That is the line integral around the region Σ which contains points where discontinuity may occur, is not altered in value if we take the integral over a new boundary which shall include no new point of discontinuity.

Again, let Σ denote a certain region, any moving point in which is given by $z=x+iy$, let this region also contain a certain fixed point $\gamma=\alpha+i\beta$, then the fraction

$$\frac{1}{z-\gamma}=\frac{1}{(x+iy)-(\alpha+i\beta)}$$

possesses the property of remaining finite and continuous throughout the region except at the point γ when it becomes infinite. The same is true if the function

$$\frac{f(z)}{z-\gamma}=\frac{f(x+iy)}{(x+iy)-(\alpha+i\beta)}.$$

If we describe around the point γ a small circle and call this σ, then the above function will be continuous and finite throughout the entire region $\Sigma-\sigma$, no matter how small the circle. Now if the boundary of Σ be considered as an enlargement of that of σ, then the integral of the above function will be the same whether it be taken in the positive direction around the boundary of σ or around that of Σ. If we designate the points on the boundary of σ by $\zeta+i\eta$, this will give us

$$\int\frac{f(\zeta+i\eta)d(\zeta+i\eta)}{(\zeta+i\eta)-(\alpha+i\beta)}=\int\frac{f(x+iy)d(x+iy)}{(x+iy)-(\alpha+i\beta)}$$

Consider a circle passing through $\zeta+i\eta$ and having its center at $\alpha+i\beta$ and radius $=\rho$, let the angle of the radius with x be ϑ, then

$$\zeta-\alpha=\rho\cos\vartheta$$
$$\eta-\beta=\rho\sin\vartheta$$

and

$$(\zeta+i\eta)-(\alpha+i\beta)=\rho(\cos\vartheta+i\sin\vartheta)=\rho\varepsilon^{i\vartheta}$$

and consequently

$$d(\zeta+i\eta)=i\rho\varepsilon^{i\vartheta}d\vartheta,$$

which gives

$$\frac{d(\zeta+i\eta)}{(\zeta+i\eta)-(\alpha+i\beta)}=id\vartheta$$

our left hand integral now becomes

$$i\int f(\zeta+i\eta)d\vartheta$$

or

$$2\pi i\int\frac{f(\zeta+i\eta)\rho d\vartheta}{2\pi\rho}$$

Representing the circumference of σ by this symbol, the integral is

$$=\frac{2\pi i}{\sigma}\int f(\zeta+i\eta)d\sigma$$

and our equation becomes

$$\frac{1}{\sigma}\int f(\zeta+i\eta)d\sigma=\frac{1}{2\pi i}\int\frac{f(x+iy)d(x+iy)}{(x+iy)-(\alpha+i\beta)}$$

Representing the integral on the left by F, allow the circle whose radius is ρ to become smaller and smaller. The function F is the mean of all the values which the function f assumes upon the circle described around $\alpha+i\beta$ as center. As the circle becomes smaller, the value of F gradually changes, approaching the value $f(\alpha+i\beta)$ which f has at the point $\alpha+i\beta$, and when the circle has so contracted as to coincide with the point F becomes$=f(\alpha+i\beta)$, giving us then

$$f(\alpha+i\beta)=\frac{1}{2\pi i}\int\frac{f(x+iy)d(x+iy)}{(x+iy)-(\alpha+i\beta)}$$

or

$$f(\gamma)=\frac{1}{2\pi i}\int\frac{f(z)dz}{z-\gamma}$$

By interchange of $\alpha+i\beta$ with $x+iy$ which a little consideration will show to be permissible, this becomes

$$f(z)=\frac{1}{2\pi i}\int\frac{f(\gamma)d\gamma}{\gamma-2}$$

when z is a point inside and γ a point on the boundary of ε. Differentiating with respect to z gives us

$$f'(z)=\frac{1}{2\pi i}\int\frac{f(\gamma)d\gamma}{(\gamma-z)^2}$$

$$f''(z)=\frac{1}{2\pi i}\int\frac{f(\gamma)d\gamma}{(\gamma-z)^3}$$

$$f^{n-1}(z)=\frac{1}{2\pi i}\int\frac{f(\gamma)d\gamma}{(\gamma-z)^n}$$

when $\frac{1}{\gamma-z}$, n being any whole number, varies continuously with the motion of z, provided that z never coincides with γ. [The same is true of the sum

$$\frac{\Gamma_1}{(\gamma_1-2)^n}+\frac{\Gamma_2}{(\gamma_2-2)_n}+\quad\ldots\quad\frac{\Gamma_n}{(\gamma_n-2)^n}$$

the numerators Γ being arbitrary constants]. Consequently follows that the derived functions $f'(z)$, $f''(z)$ . . . . or $f'(x+iy)$, $f''(x+iy)$ . . . . will be single valued and continuous inside the region Σ if the function $f(x+iy)$ is within the same region, single valued and continuous. Suppose that we have two functions f_1 and f_2 of z in the form

$$f_1=\mathrm{U}+i\mathrm{V}$$
$$f_2=\mathrm{U}'+i\mathrm{V}'$$

then will f_1f_2 be a function of z.

$$f_1f_2=\mathrm{UU}'-\mathrm{VV}'+i(\mathrm{V}'\mathrm{U}+\mathrm{U}'\mathrm{V})$$

$$\frac{d}{dx}(\mathrm{UU}'-\mathrm{VV}')=\mathrm{U}\frac{d\mathrm{U}'}{dx}+\mathrm{U}'\frac{d\mathrm{U}}{dx}-\mathrm{V}\frac{d\mathrm{V}'}{dx}-\mathrm{V}'\frac{d\mathrm{V}}{dx}$$

. . .
. . .
. . .

From these follow, by remembering the relations among these differential coefficients

$$\frac{d}{dx}(\mathrm{UU}'-\mathrm{VV}')=\frac{d}{dy}(\mathrm{UV}'+\mathrm{VU}')$$

$$\frac{d}{dx}(\mathrm{UV}'+\mathrm{VU}')=-\frac{d}{dy}(\mathrm{UU}-\mathrm{VV}')$$

which proves the theorum.

Take the differential coefficient

$$\frac{df}{dz}=\frac{d(\mathrm{U}+i\mathrm{V})}{d(x+iy)}$$

this is

$$=\frac{\left(\frac{d\mathrm{U}}{dx}+i\frac{d\mathrm{V}}{dx}\right)dx+\left(\frac{d\mathrm{U}}{dy}+i\frac{d\mathrm{V}}{dy}\right)dy}{dx+idy}$$

$$=\frac{d\mathrm{U}}{dx}+i\frac{d\mathrm{V}}{dx}$$

or $\frac{df}{dz}$ is a function of z independent of dz. In fact this is the definition that Reimann gives of the functions of complex variables. He says; "a variable complex quantity w is said to be a function of another variable complex quantity z if it so varied with z that the value of the differential coefficient $\frac{dw}{dz}$ is independent of dz."

Place

$$\mathrm{M}^2=\left(\frac{d\mathrm{U}}{dx}\right)^2+\left(\frac{d\mathrm{V}}{dx}\right)^2=\left(\frac{d\mathrm{U}}{dy}\right)^2+\left(\frac{d\mathrm{V}}{dy}\right)^2$$
$$=\left(\frac{d\mathrm{U}}{dx}\right)^2+\left(\frac{d\mathrm{U}}{dy}\right)^2=\left(\frac{d\mathrm{V}}{dx}\right)^2+\left(\frac{d\mathrm{V}}{dy}\right)^2$$
$$=\frac{d\mathrm{U}}{dx}\frac{d\mathrm{V}}{dy}-\frac{d\mathrm{U}}{dy}\frac{d\mathrm{V}}{dx}$$

From these together with the identities

$$1=\frac{dx}{d\mathrm{U}}\frac{d\mathrm{U}}{dx}+\frac{dx}{d\mathrm{V}}\frac{d\mathrm{V}}{dx},$$

$$o=\frac{dy}{d\mathrm{U}}\frac{d\mathrm{U}}{dx}+\frac{dy}{d\mathrm{V}}\frac{d\mathrm{V}}{dx},$$

$$o=\frac{dx}{d\mathrm{U}}\frac{d\mathrm{U}}{dy}+\frac{dx}{d\mathrm{V}}\frac{d\mathrm{V}}{dy},$$

$$1=\frac{dy}{d\mathrm{U}}\frac{d\mathrm{U}}{dy}+\frac{dy}{d\mathrm{V}}\frac{d\mathrm{V}}{dy},$$

we have

$$\frac{dx}{d\mathrm{U}}=\frac{1}{\mathrm{M}^2}\frac{d\mathrm{U}}{dx},\quad\frac{dy}{d\mathrm{U}}=-\frac{1}{\mathrm{M}^2}\frac{d\mathrm{V}}{dx}$$

$$\frac{dx}{d\mathrm{V}}=-\frac{1}{\mathrm{M}^2}\frac{d\mathrm{U}}{dy},\quad\frac{dy}{d\mathrm{V}}=\frac{1}{\mathrm{M}^2}\frac{d\mathrm{V}}{dy}$$

and from these also follow

$$\frac{dx}{d\mathrm{U}}=\frac{dy}{d\mathrm{V}},\quad\frac{dx}{d\mathrm{V}}=-\frac{dy}{d\mathrm{U}}$$

From these it follows that if $f'(z)$, $(=w)$, is a function of z, then inversely z is function of w. If we now consider x and y as the co-ordinates of a certain point in the plane xy we may also consider U and V to be the co-ordinates of a certain point in another plane. Now the above assumptions holding concerning w and z we have seen that $\frac{dw}{dz}$ is a single valued and continuous function of z and also $\frac{dz}{dw}$ is such a function of w. Now place

$$\frac{dw}{dz}=\mathrm{M}(\cos\Im+i\sin\Im)$$

then M and $\Im$ are functions of x and y. This equation becomes when written in full

$$d\mathrm{U}+id\mathrm{V}=\mathrm{M}(\cos\Im+i\sin\Im)(dx+idy)$$

From which by equating the real and imaginary parts separately

$$d\mathrm{U}=\mathrm{M}(\cos\Im dx-\sin\Im dy)$$
$$d\mathrm{V}=\mathrm{M}(\sin\Im dx+\cos\Im dy)$$

Now if we assume that the z plane and the w plane originally coincide with a material plane, that the axis of U coincides with that of x, the axis of V coincides with that of y, we have that x and y representing the co-ordinates of a material point of this plane, U and V represent the new co-ordinates of the same point in a certain new position of the plane. The above equations then give the variation of an element of this plane due to a certain displacement par allel to itself, a positive rotation through an angle $\Im$, and an expansion of its linear dimensions in a ratio of 1:M. We have

$$d\mathrm{U}^2+d\mathrm{V}^2=\mathrm{M}(dx^2+dy^2)$$

and it is easy to show by means of the direction cosines, that the angles between the corresponding sides of the element are equal in both positions, that is, the element of the plane in its new position is similar to the corresponding element in the original position. It is not true of a finite portion of either planes that it is similar to a finite portion of the other, except in certain special cases. When such a correspondence holds throughout the entire regions of w and z, these planes are said to be projected upon each other in similar elements. Assume now

$$w=\frac{\alpha+\beta z}{\gamma+\delta z}\text{ and }z=-\frac{\alpha-\gamma w}{\beta-\delta w}$$

$\alpha,\beta,\gamma,\delta$ being simple constants, z and w are now single valued functions the one of the other throughout the entire regions of these variables, they are not, however, everywhere continuous; since for $\gamma+\delta z=o$ we have $w=\infty$ and for $\beta-\delta w=o$ $z=\infty$. Consider the region of z to be bounded by two closed lines, one infinitely small around the point $\gamma+\delta z=o$, and the other infinitely great.

The region of w is then also limited by two closed curves, one infinitesimal corresponding to the infinitely great curve of the region z, and the other infinitely great corresponding to the infinitesimal curve around the point $\gamma+\delta z=o$. We will now show that to any circle in the region of z then corresponds some circle in the region of w. Assume two new variables z' and w' such that

$$z=a+bz',\ w=\mathrm{A}+\mathrm{B}w'$$

The constants being such that

$$w'=\frac{1}{z},$$

the constants can be determined easily when $\delta\pm o$, by observing that for $z'=o$ $w'=\infty$ and conversely, and for either of these $=1$ the other is also$=1$. We have then without difficulty

$$\beta-\mathrm{A}\delta=o$$
$$\gamma+a\delta=o$$
$$a+a\beta=\mathrm{B}b\delta$$

We have now

$$z'=x'+iy',\ w'=\mathrm{U}'+i\mathrm{V}'$$

and from the above relation between w' and z'

$$\mathrm{U}'+i\mathrm{V}'=\frac{1}{x'+iy'}=\frac{x'-iy'}{x'^2+y'^2}$$

hence

$$\mathrm{U}'=\frac{x'}{x'^2+y'^2},\ \mathrm{V}=\frac{-y}{x'^2+y'^2}$$

and

$$x'=\frac{\mathrm{U}'}{\mathrm{U}'^2+\mathrm{V}'^2},\ y'=-\frac{\mathrm{V}}{\mathrm{U}'^2+\mathrm{V}'^2}$$

If then we have between $x'y'$ the relation

$$a_1(x'^2+y'^2)+a_2x'+a_3y'+a_4=o$$

we will also have between U′ and V′ the relation

$$a_1+a_2\mathrm{U}'-a_3\mathrm{V}'+a_4(\mathrm{U}'^2+\mathrm{V}'^2)=o$$

that is, a circle in the plane of z' corresponds to a circle in the plane of w'. Now a circle in the plane of z obviously corresponds to a circle in the plane of z' and similarly for w and w', therefore finally a circle in the plane of z will correspond to a circle in the plane of w.

In the relation

$$w=\frac{a+\beta z}{\gamma+\delta z}$$

these are really only three independent

complex constants, viz.: the ratios $\alpha:\beta:\gamma:\delta$. We can establish then such linear relations among these as to make three arbitrarily chosen points a, b, c, in the z plane correspond to three such points A, B, C in the w plane, then by virtue of what has been shown above, the circle described through a, b, c will correspond to the circle though A, B, C. Designate the surfaces included within these circles by s and S respectively. These areas will correspond to each other if neither of them contain a point of discontinuity. Suppose, however, that the point $\gamma+\delta z=o$ lies within s, then the surfaces s and S do not correspond, but each corresponds to that portion of the region of the other which remains after taking away the portions s and S respectively. The point in the region of w that corresponds to the point $\gamma+\delta z=o$ lies at infinity, and if we describe about this point $\gamma+\delta z=o$, an infinitely small closed curve, it will correspond in the region of w to an infinitely great closed curve. All outside of this curve, then, is the region of z, and corresponds to the region of w included within the infinitely great closed curve.

In the expressions

$$w=\frac{\alpha+\beta z}{\gamma+\delta z},\quad z=-\frac{\alpha-\gamma w}{\beta-\delta w}$$

we saw that such relations could be introduced among the independent constants as to cause them to represent three arbitrary corresponding points in the planes of z and w, and that consequently through these points we can describe circles which shall correspond.

Then the circles which pass through two points a, b of the plane z will correspond to the circles which pass through the two corresponding points A, B of the w plane and make the same angles with each other. The surface included between these two arcs will be in the form of a crescent, and we will designate it by this name. If one of the vertices of the crescent for example the point B passes to infinity the bounding circles which pass through a will become straight lines and the figure thus formed we may approximately call a sector. The equations

$$w=\frac{\alpha+\beta z}{\gamma+dz},\qquad z=-\frac{\alpha-\gamma w}{\beta-dw}$$

now afford us the means of projecting an arbitrary crescent in the plane of z upon an arbitrary crescent in the plane of w so that the vertices a, b shall correspond to A,B and also so that a third point c, in the periphery of the z crescent may correspond to a third point C in the periphery of the w crescent, provided that the motion in the direction $abca$ shall have the same sign as the motion in the direction ABCA. We will state without proof another principle. If the region of z contains no points in which $\frac{dw}{dz}$ is either equal to zero or infinity, and if further this region is bounded by a closed non-cutting line, and if the correspoding line in the w re- is non-cutting then throughout this region of w the points correspond *singly* to those in the region of z. The bounding line of the z region may be allowed to approach indefinitely near to certain points in which $\frac{dw}{dz}$ is either zero or infinite, and in this sense we may say that such points can lie in the bounding line of the region. Assume now

$$w=(x+iy)^n=z^n$$

where n is a real constant. Also write

$$\mathrm{U}=\mathrm{R}\cos\theta \qquad x=r\cos\vartheta$$
$$\mathrm{V}=\mathrm{R}\sin\theta \qquad y=r\sin\vartheta$$

then we have

$$\mathrm{R}[\cos\theta+i\sin\theta]=\gamma^n(\cos n\vartheta+i\sin n\vartheta)$$

from which follows

$$\mathrm{R}=\gamma^n \qquad \theta+n\vartheta$$

Now if we conceive the z region to be bounded by two circles $\gamma=$const., and two right lines $\vartheta=$const. then will the w region be bounded by two corresponding circles R=const. and two corresponding right lines $\theta=$const. If one of the circles be conceived infinitely small and the other infinitely large, our two regions become sectors whose angles a and A are connected by the relation $\mathrm{A}=na$. If neither a nor A be greater than 2π then a single point in the z region will correspond to a single point in the w region.

We have also

$$\frac{dw}{dz}=nz^{n-1}$$

this is$=o$ for $z=o$ and $n>1$; for $n<1$ it is equal to infinity. Introduce now two new planes z' and w' and place

$$z=k\frac{z'-c_1}{z'-c_2}, \text{ and } w=K\frac{w'-C_1}{w'-C_2}$$

By means of these equations our two sectors which form the regions of z and w are projected upon two crescents in the planes of z' and w'. The vertices of these crescents are the points

$$z'=c_1,\ w'=C_1,$$
$$z'=c_2,\ w'=C_2.$$

The complex constants k and K can be so chosen that two arbitrary corresponding points of the peripheries of the sectors shall correspond to two arbitrary corresponding points in the peripheries of the crescents, always remembering that the motion from two corresponding points of the figures, through the vertex, to the remaining corresponding points must have the same sign. We have now, by elimination of z and w by means of the relation

$$w=z\frac{A}{a}$$

$$\left(\text{since } n=\frac{A}{a}\right)$$

$$\frac{K}{k^n}\frac{w'-C_1}{w'-C_2}=\left(\frac{z'-c_1}{z'-c_2}\right)\frac{A}{a}$$

according as A is greater or less than a will, at the vertices, be the differential coefficient $\frac{dw'}{dz'}$ be$=o$ or$=\infty$.

This equation gives us the means of projecting one crescent upon another having a different angle, so that the vertices and also two arbitrarily chosen points in the peripheries shall correspond. Assume now that $A=\pi$ that is the crescent in the w' plane is a complete circle. Our above equation becomes, by omitting the now unnecessary accents and writing $\frac{K}{k^n}=N$,

$$N\frac{w-C_1}{w-C_2}=\left(\frac{z-c_1}{z-c_2}\right)\frac{\pi}{a}$$

Now if the constants N, C_1, C_2 be so chosen that three arbitrary points of the z crescent shall correspond to three arbitrary points in w plane, then this equation gives the projection of our z crescent upon an entire circular in the w plane, whose circumference passes through these three points. Now introduce again a new variable z', and conceive in the plane of z' that we have a crescent whose vertices are

$$z'=c_1'=c_2'$$

and whose angle$=a'$. Now if we write

$$N'\frac{w-C_1'}{w-C_2'}=\left(\frac{z'-c_1'}{z'-c_2'}\right)\frac{\pi}{a'}$$

and determine the constants N', C_1', C_2', so that three arbitrary points of the periphery of this z crescent shall correspond to the three points already determined in the w plane as corresponding to three points of the z crescent, then are these crescents in z and z' planes both projected upon the same circle in the w plane; and also are they projected upon each other so that three arbitrary points in the periphery of the one correspond to three arbitrary points in the periphery of the other. We can obtain the equation between z and z' by eliminating w from the two equations which contain these quantities. If both vertices of a crescent pass to infinity, our crescent will assume the form of a plane strip with parallel sides, this figure we will simply call a *strip*. If we write

$$-c_1=c_2=$$
$$=l(\cos\lambda+i\sin\lambda)$$

we see that l designates the half distance between the vertices of the z crescent, and λ the angle which this connecting line makes with the axis of x. If the lengths of the circular arcs that bound the crescent be denoted in parts of the radii by 2β and 2γ, then clearly the angle a between them will be given by

$$\gamma-\beta=a$$

Call the greatest breadth of the crescent b. Then we easily see that $l\tan\frac{\gamma}{2}$, and $l\tan\frac{\beta}{2}$ represent the attitudes of the two arcs measured from the line connecting their intersections, and consequently

$$b=l\left(\tan\frac{\gamma}{2}-\tan\frac{\beta}{2}\right)$$

Now when the crescent becomes a strip, γ and β become infinitely small, and $\tan\frac{\gamma}{2}$, $\tan\frac{\beta}{2}$ sensibly equal to $\frac{a}{2}$ which is of course also infinitely small; l, how-

ever, becomes infinitely great, and we now have

$$b=\frac{la}{2}.$$

Now by omitting the factor $(--)^{\frac{\pi}{a}}$ the quantity

$$\left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{a}} \text{ becomes } \left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{a}}$$

and this by replacing the infinitely great quantities c_1 and c_2 by c, becomes

$$=\left(1+z\frac{z}{c}\right)^{\frac{\pi}{a}}=\left(1+\frac{a}{\pi}\frac{z\pi}{ac}z\right)^{\frac{\pi}{a}}$$

Now for $a=o$ and $ac=$const. we have developing by the Binomial Theorum

$$\left(\frac{z-c_1}{c_2-z}\right)^{\frac{\pi}{a}}=l^{\frac{2\pi z}{ac}}=l^{\frac{\pi z}{b}}(\cos\lambda-i\sin\lambda)$$

since

$$b=\frac{la}{z}=\frac{ac(\cos\lambda+i\sin\lambda)^{-1}}{z}$$

$$=\frac{ac}{z}(\cos\lambda-i\sin\lambda).$$

We will return now to the remark made in the chapter when we showed that the conjugate functions U and V equated to constants represented orthogonal loci, and that consequently the equations

$$U=\text{const. and } V=\text{const.}$$

might represent respectively the equations of the equi-potential surfaces and the equation of the stream lines. As a simple example let us take

$$U+iV=\log\frac{(x+iy)-(a_1+ib_1)}{(x+iy)-(a_2+ib_2)}$$

or simply

$$w=\log\frac{z-c_1}{z-c_2}$$

write also

$$x-a_1=\gamma\cos\vartheta_1, \qquad x-a_2=\gamma\cos\vartheta_2$$
$$y-b_1=\gamma\sin\vartheta_1, \qquad y-b_2=\gamma\sin\vartheta_2$$

Then follows easily

$$U=\log\frac{\gamma_1}{\gamma_2}, \qquad V=\vartheta_1-\vartheta_2$$

Now for the stream lines write V=const. that is

$$\vartheta_1-\vartheta_2=\text{const.}$$

Now $\vartheta_1-\vartheta_2$ is the angle made with each by the radii γ_1 and γ_2 drawn between the points whose co-ordinates are (x,y,a_1,b_1) and (x,y,a_2,b_2) or (z,c_1) and (z,c_2). The condition that this angle shall be constant gives that the point x,y lies on the circumference of a circle passing through the points c_1 and c_2. The arcs of circles then which join these points are the stream lines. For the equipotential surfaces we have $\frac{\gamma_1}{\gamma_2}=$const., and it is easy to see that this represents a circle passing through two points on the line c_1c_2 which are harmonically situated with respect to the points c_1 and c_2. We have from the equation

$$w=U+iV$$

by differentiation with respect to z

$$\frac{dw}{dz}=\frac{dU}{dx}+i\frac{dV}{dx}$$

$$=\frac{dU}{dx}-i\frac{dV}{dy}$$

and also

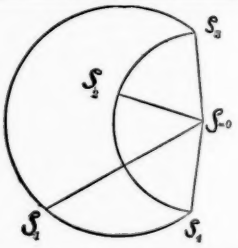
$$\frac{dz}{dw}=\frac{1}{\sqrt{\left(\frac{dU}{dx}\right)^2+\left(\frac{dU}{dy}\right)^2}}\frac{\frac{dU}{dx}+i\frac{dU}{dy}}{\sqrt{\left(\frac{dU}{dx}\right)^2+\left(\frac{dU}{dy}\right)^2}}$$

Calling this quantity ζ we may write

$$\frac{dz}{dw}=\zeta=\xi+i\eta=\rho(\cos\vartheta+i\sin\vartheta)$$

When ξ and η are the rectangular co-ordinates of a point in a plane that we may call the ζ plane, and so taken that the axes of ζ and x are parallel as also those of η and y. Then we see that ρ denoting a line drawn from the origin of the new system of axes to the point ζ that $\frac{1}{\rho}$ is =to the velocity, and the direction of ρ is the direction of the motion in the point z. For example suppose that the region of ζ is a crescent as shewn in the figure, and that during a certain portion of the fluid motion the velocity is constant and =1. Then the point $\zeta=o$ may be taken as the center of the inner arc of the crescent and this radius equal to unity.

The velocity will be constant in every portion of z that corresponds to a point on the inner circle of the crescent. At the points ζ_3 and ζ_4, however, there will



be a change both in the amount and direction of the velocity. And it is easy to see that the points in the region of z corresponding to these points, will be points of contra-flexure in the stream lines. But we will examine this more closely again.

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If we have no external forces acting on the fluid, the equation which gives the fluid pressure, as obtained in the first chapter, becomes simply, after integration making the density $\rho=1$,

$$p=C-\frac{1}{2}\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2+\left(\frac{d\varphi}{dz}\right)^2\right\}$$

The quantity in the brackets represents the square of the velocity. Then we see that as the pressure diminishes the velocity increases, the former becoming equal to $-\infty$ for the latter $=+\infty$. In a fluid we know however that an indefinite diminution of the pressure is not possible, for after having reached a certain point the fluid must fly apart. Now suppose the pressure to have reached this value or, say, suppose the pressure to assume this value then there will be a sudden increase in the velocity. This is evidently the case of a fluid issuing from an orifice: there is a sudden change in the velocity owing to their being no longer a pressure normal to the bounding of the stream. This bounding surface through which there is no flow can be given, as before mentioned, by the equation V=const. since it is made up of stream lines. The surface where the velocity suddenly alters is the surface which separates one fluid from another; the pressure and normal components of the velocity must be equal upon opposite sides of this surface. Suppose that we have only two fluids, one which occupies a certain region, in motion and is in connection with another which is at rest. The surface where this occurs is then composed of stream lines and the velocity in this surface is constant throughout the entire extent, and, if we please, $=1$. We further assume now in accordance with the assumption of the previous chapter, that the velocity potential U is a function only of x and y, and that we have

$$z=x+iy \qquad w=U+iV.$$

$$\frac{dz}{dw}=\zeta=\xi+i\eta=\rho(\cos\vartheta+i\sin\vartheta),$$

Now our problem is as follows. Regarding U and V as rectangular co-ordinates of a point in the w plane, we make certain suppositions concerning the *regions* of ζ and w and then seek to find the conditions which must exist between these quantities that the regions may be projected upon each other in such a way that their elements shall be similar. Having found this relation we have z from the equation

$$z=\int\zeta dw.$$

this gives then the region of z corresponding to the assumed regions of ζ and w, and so determines the form of the space occupied by the moving fluid. We evidently solve an inverse problem, assuming the motion and finding the form of the vessel containing the fluid, and also the position and form of the orifice. After the fluid passes the orifice the velocity is constant, so that from what we found concerning ρ, we can represent the velocity at any point outside of the orifice in magnitude and direction, by equal lines drawn from the point $\zeta=o$ to other points in the region of ζ. These points will clearly lie in a circle whose center is at $\zeta=-o$, and this circular arc will correspond to the boundaries of the fluid outside of the orifice, or what we may call the *free boundaries*. Now if we draw another circle, cuttting this first and take one of the crescents so formed for the region of ζ we will have that the radii vectores drawn to the different points of this region will represent in magnitude and direction the velocity at the corresponding point of the z region. Assume now that our w region is a certain strip at the limits of which the following equations hold:

$$V=V_0 \qquad U=-\infty$$
$$V=V_0+a \quad U=+\infty$$

a and V_0 being constants. Now suppose that the region of ζ is a crescent one of whose limiting circles has for equation

$$\rho=1$$

and for every point in the interior of the crescent $\rho>1$. The velocity then, which varies as $\frac{1}{\rho}$, diminishes as we travel along the outer circle of the crescents, becoming at the vertices=1, where its variation suddenly becomes $=o$. As the direction of motion of the radius vector here also changes suddenly into the opposite, we see as before mentioned, that the points, where the fluid leaves the orifice, are points of contra-flexure on the corresponding stream lines. We are at liberty now to choose three points in the limits of our crescent to correspond to three points in the limits of the strip. Assume then that

$$\zeta=\zeta_1 \text{ for } U=-\infty$$

and

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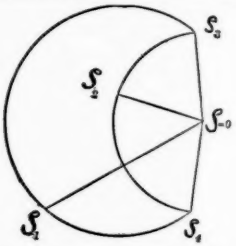
$$\frac{dz}{dw}=\rho(\cos\vartheta+i\sin\vartheta)$$

we see that all linear dimensions of the elementary portions of the z region are enlarged in the projection upon the w region in the ratio of $1:\rho$. In the neighborhood of z_2, that is, when we have $\rho=\rho_2=1$, this ratio becomes=1, consequently the path of the z region is congruent to the corresponding part of the w region. Now b is the breadth of the stream at this point, then at z_1 the breadth is $\rho_1 b$. In z_2, and at finite distances from it, the velocity is constant $=1$, then the stream has a free boundary line. In z_1, and at finite distances therefrom, the velocity is variable, and proportional to $\frac{1}{\rho_1}$ that is, is less than at z_2, this portion of the fluid region is then bounded by fixed walls. Calling ζ_3 and ζ_4 the vertices of the ζ crescent, then the corresponding points of the z region may be called z_3 and z_4 as for these points on the crescent, the velocity suddenly changes from variable to constant, so for the corresponding points in the z region will the boundaries change from fixed to free. As the radius vector ρ travels from ζ_3 to ζ_1 and here becomes parallel to an infinite line, so in moving from z_3 along the boundary to z_1, do we traverse a curve the tangents to which are parallel to the radii vectores to the corresponding points in the ζ crescent, until we arrive at the tangent parallel to ρ_1 which extends to infinity, when suddenly changing, we travel down the opposite boundary of the z region, ρ_1 occupying still the same position, and then our boundary of the z region begins again to be curved, and continues so to curve until we reach z_4, which with z_3 is, as has already been mentioned, a point of contra-flexure. The curvature continues after this in the opposite direction as we follow the circle $\rho=1$, until we come again to ρ_2, when the same operation is repeated that we described as going on in the region bounded by fixed walls. Call ds an element of a stream line. Then we have for the velocity



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$$\frac{dU}{ds}=\frac{1}{\rho} \text{ or } ds=\rho dU.$$

Now we defined an angle ϑ by the equation

$$\zeta=\xi+i\eta=\rho(\cos\vartheta+i\sin\vartheta)$$

And for the radius of curvature of ds we have $\frac{ds}{d\vartheta}$

$$=\frac{\rho dU}{d\vartheta}$$

Now we have $w=U+iV$, and for stream lines V=const., therefore in this case is

$$dU=dw$$

and the radius of curvature is

$$=\frac{\rho dw}{d\vartheta}$$

Now our above equation giving ζ is evidently equivalent to

$$\zeta=\rho\varepsilon^{i\vartheta}$$

Taking logarithms of this

$$\log\zeta=\log\rho+i\vartheta$$

differentiating

$$\frac{d\zeta}{\zeta}=\frac{d\rho}{\rho}+id\vartheta$$

If the stream line is one lying in the free limits of the stream, then $\rho=1$ and $d\rho=o$ consequently

$$d\vartheta=-i\frac{d\zeta}{\zeta}$$

and the radius of curvature becomes

$$=i\zeta\frac{dw}{d\zeta}$$

We suppose the angle of the crescent to $<\pi$; then the portions of the ζ region infinitely near the points ζ_3 and ζ_4 we may regard as portions of sectors which by the relation $\frac{dz}{dw}=\zeta$ are projected upon corresponding sectors in the w region, and consequently at those points the ratio $\frac{dw}{d\zeta}$ will be indefinitely small, and so by the above equation will be the radius of curvature. For any point of the fixed walls $d\rho\neq o$. We can by differentiation of the equation between ρ and ϑ which expresses the second circle of the ζ crescent, obtain a relation between $d\rho$ and and $d\vartheta$, then by aid of the equation

$$\frac{d\zeta}{\zeta}=\frac{d\rho}{\rho}+id\vartheta$$

we can express $d\vartheta$ as function of $d\zeta$. If this circle becomes a right line its equation will be ϑ=const. or $d\vartheta=o$. But in general we see that we can express the radius of curvature by an expression containing as a factor the quantity $\frac{dw}{d\zeta}$ which vanishes as the point z passes through z_3 or z_4.

Suppose that the orifice under consideration is simply an opening in a plane wall; we may assume the wall as horizontal and the axis of x lying in the plane that of y perpendicular to the wall. Our ζ region will evidently be formed by a crescent, a complete semi-circle for which $\rho=1$ corresponding to the free boundaries of the fluid, and an infinite line corresponding to the fixed wall; the angle of the crescent being $=\frac{\pi}{2}$. The points ζ_3 and ζ_4 are of course the vertices of the crescent and correspond as before to the points z_3 and z_4 which are at the intersection of the free and fixed boundaries. The point ζ_2 lies midway between ζ_3 and ζ_4 on the semi-circle of the crescent; and ζ_1 lies at infinity on the line $\zeta_2\zeta_3$ produced. For the w region we have the same values of U given before and for the corresponding values of V assume

$$V=o, \text{ and } V=\pi.$$

In the preceding chapter we saw that we could express one of the quantities

$$\left(\frac{z'-c_1'}{z'-c_2'}\right)^{\frac{\pi}{\alpha'}},\quad \left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{\alpha}}$$

as a function of the other and constants by eliminating w from the equations

$$N\frac{w-C_1}{w-C_2}=\left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{\alpha}}$$

$$N'\frac{w-C_1'}{w-C_2'}=\left(\frac{z'-c_1'}{z'-c_2'}\right)^{\frac{\pi}{\alpha'}};$$

the result of the elimination is

$$\left(\frac{z'-c_1'}{z'-c_2'}\right)^{\frac{\pi}{\alpha}}=N'\frac{N\frac{(C_1-C_1')}{(C_2+C_2')}+\left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{\alpha}}}{N\frac{C_1-C_2'}{C_2+C_2'}+\left(\frac{z-c_1}{z-c_2}\right)^{\frac{\pi}{\alpha}}}$$

and we also find

$$\left(\frac{z-c_1}{c_2-z}\right)^{\frac{\pi}{a}} = l^{\frac{\pi 2}{b}}(\cos\lambda - i\sin\lambda)$$

Now in this case write $\zeta = z'$ then $a' = \frac{\pi}{2}$; and from the last equation by writing $z = w$ gives for the second member l^w since $\lambda = o$ and $b = \pi$ the breadth of the stream. The above equation now becomes simply, since $c_1' = 1$ and $c_2' = -1$

$$\left(\frac{\zeta-1}{\zeta+1}\right)^2 = \mathrm{K}.\frac{\varepsilon^w - \mathrm{C}}{\varepsilon^w - \mathrm{C}'}$$

where the three constants K, C, C′, have to be determined according to the given conditions of the problem. The radius vector ρ is $=\infty$ for the infinite values of U; further, the line ρ_1, *i.e.*, from ζ_0 to ζ_1, is drawn in the negative direction of the axis of η, and consequently $\vartheta = \frac{3\pi}{2}$ and $\sin\vartheta = -1$, $\cos\vartheta = o$. We therefore have for $\mathrm{U} = -\infty$

$$\zeta_1 = -i\infty$$

and for $\mathrm{U} = +\infty$, $\rho = 1$ and

$$\zeta_2 = -i$$

We are at liberty, however, to assume another condition which we will by the equations

$$\zeta = 1 \text{ for } w = o$$

these give us readily $\mathrm{K} = -1$, $\mathrm{C} = 1$, $\mathrm{C}' = -1$, and our equation becomes

$$\left(\frac{\zeta-1}{\zeta+1}\right)^2 = \frac{1-\varepsilon^w}{1+\varepsilon^w}$$

Clearing of fractions

$$(\zeta-1)^2(1+\varepsilon^w) - (\zeta+1)^2(1-\varepsilon^w) = o$$

divide through by $2\varepsilon^w$

$$(\zeta-1)^2\frac{(\varepsilon^{-w}+1)}{2} - (\zeta+1)^2\frac{(\varepsilon^{-w}-1)}{2} = o$$

that is

$$\zeta^2 - 2\zeta\varepsilon^{-w} + 1 = o$$

which gives

$$\zeta = \varepsilon^{-w} + \sqrt{\varepsilon^{-2w}-1}$$

The negative sign cannot be used before this radical, because we must have $\zeta = \infty$ for U that is $w = \infty$. Now for the determination of z we have the integral

$$z = \int \zeta dw = \int (\varepsilon^{-w} + \sqrt{\varepsilon^{-2w}-1})dw$$

If we choose the lowest limit as $w = \infty$, then the point $z = o$ evidently corresponds to $\zeta = 1$, and thus the point z_1. To integrate this write

$$\sqrt{\varepsilon^{-2w}-1} = \varphi$$

then

$$\sqrt{\varphi^2+1} = \varepsilon^{-w}$$

and our integral becomes

$$z = -\int_o^{\varphi} [\sqrt{\varphi^2+1} + \varphi]\frac{\varphi d\varphi}{\varphi^2+1}$$

this is easily found to become

$$z = 1 - \varepsilon^{-w} + \sqrt{\varepsilon^{-2w}-1} + \tan^{-1}\sqrt{\varepsilon^{-2w}-1}$$

Now suppose $\mathrm{V} = o$, $o < \mathrm{U} < -\infty$ the sign of inequality referring to absolute magnitude, then as there is no imaginary part

$$x = 1 - \varepsilon^{-\mathrm{U}} - \sqrt{\varepsilon^{-2\mathrm{U}}-1} + \tan^{-1}\sqrt{\varepsilon^{-2\mathrm{U}}-1}$$
$$y = o$$

the circular function being, since the radical is positive, an arc in the first quadrant. This simply represents the negative axis of x which is a fixed boundary, is in fact a portion of the wall through which the fluid flows.

For $\mathrm{U} = -\infty$ $\mathrm{V} = o$,

$$x = 1 - 2\varepsilon^{-\mathrm{U}} + \frac{\pi}{2}$$
$$y = o$$

and for $\mathrm{U} = -\infty$ and V between o and π

$$x = 1 - 2\varepsilon^{-\mathrm{U}}\cos\mathrm{V} + \frac{\pi}{2}$$
$$y = \quad 2\varepsilon^{-\mathrm{U}}\sin\mathrm{V}$$

from these

$$\left\{x - \left(1 + \frac{\pi}{2}\right)\right\}^2 + y^2 = 4\varepsilon^{-2\mathrm{U}}$$

the equation of a circle whose center is $y = o$ $x = 1 + \frac{\pi}{2}$, and whose radius is $2\varepsilon^{-\mathrm{U}}$. The velocity through the center of this circle is $\frac{1}{\zeta} = \frac{1}{2\varepsilon^{-\mathrm{U}}}$, as is seen from the value given for ζ, making therein $w = \mathrm{U}$. For $\mathrm{U} = -\infty$, $\mathrm{V} = \pi$

$$x = 1 + z\varepsilon^{-\mathrm{U}} + \frac{\pi}{z},$$
$$y = o,$$

For U between $-\infty$ and o, and $\mathrm{V} = \pi$

$$x = 1 + \varepsilon^{-\mathrm{U}} + \sqrt{\varepsilon^{-2\mathrm{U}}-1} + \pi - \tan^{-1}\sqrt{\varepsilon^{-2\mathrm{U}}-1}$$
$$y = o.$$

This is the portion of the axis of x between $x=2+\pi$ and $x=+\infty$. It is easily seen that the point $x=2+\pi$, $y=o$ is the point z_3.

If $V=\pi$ and $U>o$, we find without difficulty since

$$\tan^{-1} i\varphi = z' - \frac{1}{z}\log\frac{1+\varphi}{1-\varphi}$$

$$x = 1+\varepsilon^{-U}+\pi$$

$$y = \sqrt{1-\varepsilon^{-U}} - \frac{1}{z}\log\frac{1+\sqrt{1-\varepsilon^{2-U}}}{1-\sqrt{1-\varepsilon^{-2U}}}$$

This line is one of the free limits of the stream. For $V=\pi$ and $U=+\infty$

$$x = 1+\pi$$

$$y = 1-\log 2-U$$

Now if V lies between π and o

$$x = 1+V$$

$$y = 1-\log 2-U$$

These give a line parallel to x of the length π and for which $y=\infty$. The fluid flows through this line in the direction of the negative axis of y with a velocity $=1$. We get the same forms of equations $V=o$ and $U>o$ as in the case of $V=\pi$ and $U>o$.

Assume now that the region of ζ is limited by an arc of radius $=1$ and of length $=a$. The remaining boundaries being right lines through the points $\zeta_1\zeta_3$ and $\zeta_2\zeta_4$, and an infinitely great circle. Then by what we have seen in a similar case in the preceding chapter if we have a ψ region formed of a semi-circle of radius $=1$, and the lines drawn from its center to its extremities produced to infinity, and an infinitely great circle, we can project our ζ region upon this ψ region by the relation

$$\psi = \zeta^{\frac{\pi}{a}}$$

We have obviously

$$\psi_3 = \zeta_3^{\frac{\pi}{a}}$$

$$\psi_4 = \zeta_4^{\frac{\pi}{a}}$$

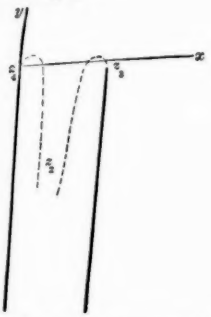
and for the projection of this ψ region upon the w region,

$$\left(\frac{\psi-\psi_3}{\psi-\psi_4}\right)^2 = K\,\frac{\varepsilon^w - C}{\varepsilon^w - C'}$$

or for the projection of the new ζ region

$$\left\{\frac{\zeta^{\frac{\pi}{a}} - \zeta_3^{\frac{\pi}{a}}}{\zeta^{\frac{\pi}{a}} - \zeta_4^{\frac{\pi}{a}}}\right\}^2 = K\,\frac{\varepsilon^w - C}{\varepsilon^w - C'}$$

The same remarks that we have previously made concerning the relation of the quantities ρ, ζ, z will also apply here. The fixed walls will be inclined to each other and parallel as before to ρ_3 and ρ_4. Suppose that $a=2\pi$, $\vartheta_3=\vartheta_4=i$, $\zeta_3=-i$ and that for $w=o$, $\zeta=i$ and $z=o$. The z region will then be of the form shown in the figure



Our equation now assumes the form

$$\frac{\zeta^{\frac{1}{2}} - \zeta_3^{\frac{1}{2}}}{\zeta^{\frac{1}{2}} - \zeta_4^{\frac{1}{2}}} = K\,\frac{\varepsilon^w - C}{\varepsilon^w - C'}$$

The quantity ζ in passing from ζ_3 to ζ_4, which are equal, must vary continuously, and it is easy to see that $\sqrt{\zeta_3} = -\sqrt{\zeta_4}$. We have also the conditions

for	$U=-\infty$,	$\zeta=\infty$
	$V=+\infty$	$\zeta=-i$
	$w=o$	$\zeta=i$

This last determines the values of the constants K, C, C', and the equation gives

$$\left(\frac{\sqrt{\zeta}-\sqrt{i}}{\sqrt{\zeta}+\sqrt{i}}\right)^2 = \frac{1-\varepsilon^w}{1+\varepsilon^w}$$

The quadratic for ζ is easily found to be

$$\zeta^2 - 2\zeta i(2\varepsilon^{-2w}-1) - 1 = o$$

From this

$$\zeta = i(2\varepsilon^{-2U} - 1 + 2\varepsilon^{-2w}\sqrt{\varepsilon^{-2w}-1})$$

We get again by simple integration

$$z = -i(\varepsilon^{-2w} + w - 1 + e^{-w}$$

$$\sqrt{\varepsilon^{-2w}-1}-\log(\varepsilon^{-w}+\sqrt{\varepsilon^{-2w}-1})$$

If $V=o$ and U between o and $-\infty$ we have since there is no real part

$$x=o$$
$$y=-(\varepsilon^{-2U}+U-1+\varepsilon^{-U}$$
$$\sqrt{\varepsilon^{-2U}-1}-\log(\varepsilon^{-U}+\sqrt{\varepsilon^{-2U}-1})$$

The equation of the negative axis of y. For U a constant infinite and negative value and V lying between o and π,

$$x=-2\varepsilon^{-2U}\sin 2V+2V$$
$$y=-2\varepsilon^{-2U}\cos 2V-2U+\frac{3}{2}+\log 2.$$

For $V=\pi$ and U negative

$$x=2\pi$$
$$y=-(\varepsilon^{-2U}+U-1+\varepsilon^{-U}$$
$$\sqrt{\varepsilon^{-2U}-1}-\log(\varepsilon^{-U}+\sqrt{\varepsilon^{-2U}-1})$$

the same form of equation as we obtained above when V was $=o$ these are the two fixed walls of the fluid, and are at a distance apart $=2\pi$. The breadth of the stream after it becomes parallel to these walls is $=\pi$.

Suppose now that we have a w region which is limited by the lines

$$V=-\frac{\pi}{2},\ U=-\infty$$

$$V=-\frac{\pi}{2},\ U=+\infty$$

and another line between these for which

$$V=o \text{ and } U>o$$

neglecting this latter limit for a moment, and we have that a strip of this form will be projected on the plane of ψ in a circle by the equation

$$e^{w}=\frac{1+\psi}{1-\psi}$$

and giving the relations here

$$\text{for } U=-\infty,\ \psi=-1$$
$$U=+\infty,\ \psi=+1$$
$$w=i\frac{\pi}{2},\quad \psi=i$$

The radius of the circle=1, its center is at $\psi=o$, and so corresponds to $w=o$. Now from these conditions we see that for $V=o$ and $U>o$, ψ will be real and <1. The line then $V=o$, $U>o$ corresponds to the radius from $\psi=o$ to $\psi=1$. The w region will then be projected upon the ψ plane in a region that is limited by this circle, and both sides of the radius for $\psi=o$ to $\psi=1$. We can again project this on a ψ' plane in a semicircle or radius $=1$ by the relation

$$\psi=\psi'^{2}$$

and also upon a crescent whose angle is $=\frac{\pi}{2}$, at the vertices of which is $\psi=\pm 1$. So then finally our w region is projected upon this crescent by means of the relation

$$e^{w}=\frac{1+\psi'^{2}}{1-\psi'^{2}}$$

As the limits of the ζ region assume a semicircle of radius 1 and an infinitely great circle cutting it at right angles. We are at liberty now to take three points ζ_1, ζ_2, ζ_3, so that

$$\text{for}\quad U=-\infty\ \zeta=\zeta_1,$$
$$V<o, U=+\infty\ \zeta=\zeta_2,$$
$$V>o, U=+\infty\ \zeta=\zeta_3;$$

we can now determine the z region.

First a stream appears flowing from infinity of a breadth$=\pi$, and parallel to the radius vector ρ. This flows to infinity in two streams parallel respectively to ρ_2 and ρ_3, and of width each$=\frac{\pi}{2}$. This is evidently the case of a stream meeting an obstacle and flowing around it; in this case the obstacle is a plane wall. The outer limits of the stream are lines whose tangents are at every point parallel to the radii vectores drawn from $\zeta=o$ to every point of $\zeta_1\zeta_2$, and $\zeta_1\zeta_4$. The inner boundaries are not wholly free, but are partially free and partially fixed, the fixed part being the wall on which the stream impinges. The ends of this plane wall correspond to the vertices of the ζ crescent. There is a certain point on this surface for which corresponds to $w=o$, and consequently when the velocity$=o$ and here a stream line splits, one part going to the right the other to the left.

Let now the region of w, being the infinite planes lying on both sides of the line, for which

$$U>o \text{ and } V=o$$

we have

$$\text{for } U=\infty,\quad \zeta=-i$$
$$w=1,\quad \zeta=\pm 1$$

From the conditions for the w region we

see that the point $w=1$ will occur twice, and consequently two different points will correspond in the ζ region. The w plane can be projected upon the ψ plane by the relation

$$w=\psi^2$$

and then

$$\begin{aligned} \text{for } w&=\infty \quad \psi=\infty \\ w&=1 \quad \psi=\pm 1 \end{aligned}$$

This ψ region is a circle of infinite radius the ζ crescent can be projected upon it by

$$\frac{(1-\zeta)^2}{(1+\zeta)^2}=\frac{1-\psi}{1+\psi}$$

and so that for

$$\begin{aligned} \zeta&=-i \quad \psi=\infty \\ \zeta&=+1 \quad \psi=+1 \\ \zeta&=-1 \quad \psi=-1 \end{aligned}$$

Therefore for the relation between ζ and w we have

$$\left(\frac{1-\zeta}{1+\zeta}\right)=\frac{1-\sqrt{w}}{1+\sqrt{w}}$$

which gives for ζ the quadratic

$$\zeta^2-\frac{2}{\sqrt{w}}\zeta+1=o$$

or

$$\zeta=\frac{1}{\sqrt{w}}-\sqrt{\frac{1}{w}-1}$$

as $\zeta=-i$ for $w=\infty$ the second radical must have the negative sign. Assume that z and w vanish together, and we have from the integral

$$z=\int\left\{\frac{1}{\sqrt{w}}\sqrt{\frac{1}{w}-1}\right\}dw$$

the value of z, viz:

$$z=2\sqrt{w}+w\sqrt{\frac{1}{w}-1}+\sin^{-1}\sqrt{w}$$

The integration is easily effected by taking

$$\sqrt{\frac{1}{w}-1}=\varphi$$

for the variable. Here $\sin^{-1}\sqrt{w}=o$ for $w=o$. The fixed wall on which the stream impinges has as we saw $V=o$, so for it $\sqrt{w}$ is real and lies between -1 and $+1$. We must have for this wall, since there is no imaginary, $y=o$, and also x lying between

$$-\left(2+\frac{\pi}{2}\right) \text{ and } +\left(2+\frac{\pi}{2}\right)$$

The breadth of the wall is then $4+\pi$.

A much more complete investigation of this problem of the flow of fluids through orifices, may be expected in a future chapter. The method of conjugate functions which has been here employed, and which has been taken largely from Kirchhoff's "*Vorlesungen über Math. Phy.*" is beautiful, and from a purely theoretical point of view, very valuable; but in the study of particular cases of fluid motion, it is necessary to take into account the form and position of the orifice, and then to determine the form of the stream, amount of contraction, and the motions of the fluid particles among themselves. Furthermore, it would be a very difficult process to assign *a priori* the forms of the ζ and w regions so as to determine the nature of the motion in and the form of the z region.

Concerning the pressure p, we have our equation for determining p in the form now

$$p=C-\tfrac{1}{2}\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2\right\}$$

this φ of course corresponding to the quantity that we have designated by U. The velocity being equal to $\frac{1}{\rho^2}$, this equatian is simply

$$p-C-\frac{1}{2}\cdot\frac{1}{\rho^2}$$

For the fluid in a state of rest the pressure is of course constant, say $=\pi$; in this case we have also $\rho=1$, and the equation becomes

$$\pi=C-\tfrac{1}{2}$$

giving

$$C=\pi+\tfrac{1}{2}$$

and thus finally for p

$$p=\pi+\tfrac{1}{2}\left(1-\frac{1}{\rho^2}\right)$$

The pressure on an element ds of a fixed wall that is in contact on one side with the fluid at rest, and on the other with the moving fluid is

$$(p-\pi)ds=\tfrac{1}{2}\left(1-\frac{1}{\rho^2}\right)ds.$$

Now we have for ds the value $\rho d\mathrm{U}$, and since at this wall $V=o$, $d\mathrm{U}=dw$ and $ds=\rho dw$. The pressure then on the element ds is

$$=\tfrac{1}{2}\left(\rho-\frac{1}{\rho}\right)dw$$

For a plane wall parallel to x ζ is real and $\rho=\pm\zeta$ the upper or lower sign according as ζ is positive or negative, since ρ is positive. Then for the total pressure on this wall we have

$$\int \pm\tfrac{1}{2}\left(\zeta-\frac{1}{\rho}\right)dw$$

as nowhere on the wall is the resultant pressure $<o$, we must choose the upper or lower sign, so that the quantity under the integral sign shall be positive. If our wall be as in the last problem discussed, we will have from the equation

$$\zeta=\frac{1}{\sqrt{w}}-\sqrt{\frac{1}{w}-1}$$

$$\tfrac{1}{2}\left(\zeta-\frac{1}{\zeta}\right)=\sqrt{\frac{1}{w}-1}$$

Now $\sqrt{w}$ increases from -1 to $+1$ integrating then and we have the pressure in this case $=\pi$. If instead of w we write $\frac{w}{n}$ when n is a positive constant, the equations which represented any particular fluid motion will now represent one in which the stream lines are the same as before, but the velocity is now throughout proportional to n, and the pressure upon any wall proportional to n^2. If both z and w be replaced by $\frac{z}{m}$, and $\frac{w}{m}$, the stream lines and velocities will both be as before, and the pressure proportional to m; the linear dimensions of the stream lines are also proportional to w. If now we again call the density of the fluid ρ, the pressure upon any wall will be proportional to ρ. If we call l the length of the wall that was met by the stream of infinite breadth, flowing in the negative direction of y, and v the velocity of the fluid, the pressure upon this wall will be

$$=\rho v^2\cdot\frac{l\pi}{4+\pi}$$

$\pi\rho v^2$ being the pressure upon unit of length, which unit is $=4+\pi$.

LINKAGES.

A KINEMATIC DISCUSSION OF THE DIFFERENT FORMS OF ARTICULATED LINKS WITH ESPECIAL REFERENCE TO PEAUCELLIER'S CELL.

By J. D. C. DE ROOS.

Translated from "Revue Universelle des Mines" for Van Nostrand's Magazine.

IV.

The other movements of the system (Figs. 24 and 25) are all circular with the exception of that of the point D relative to CB′ or C′B′ which we will not discuss here.

In the arrangement of linkages, which we will proceed to describe, and which may be termed *conicographs*, the origin is at the same time at the fixed point and at the focus or summit of the curve.

For the ellipse and hyperbola the fixed point may be placed at the center and the construction becomes more elegant.

We will couple for this purpose two of the transformed Peaucellier Cells (éléments variés) OCLANC′ and BPNAL′P′ as is shown in Fig. 36.

The sides of the lozenges of the two elements are equal, and since the links AL and AL′ are constantly in prolongation, each of the other, they can be replaced by the single bar LL′, which will render AN superfluous.

This combination of eight links enjoys the property, that the sum of the squares of OA and AB is constant.

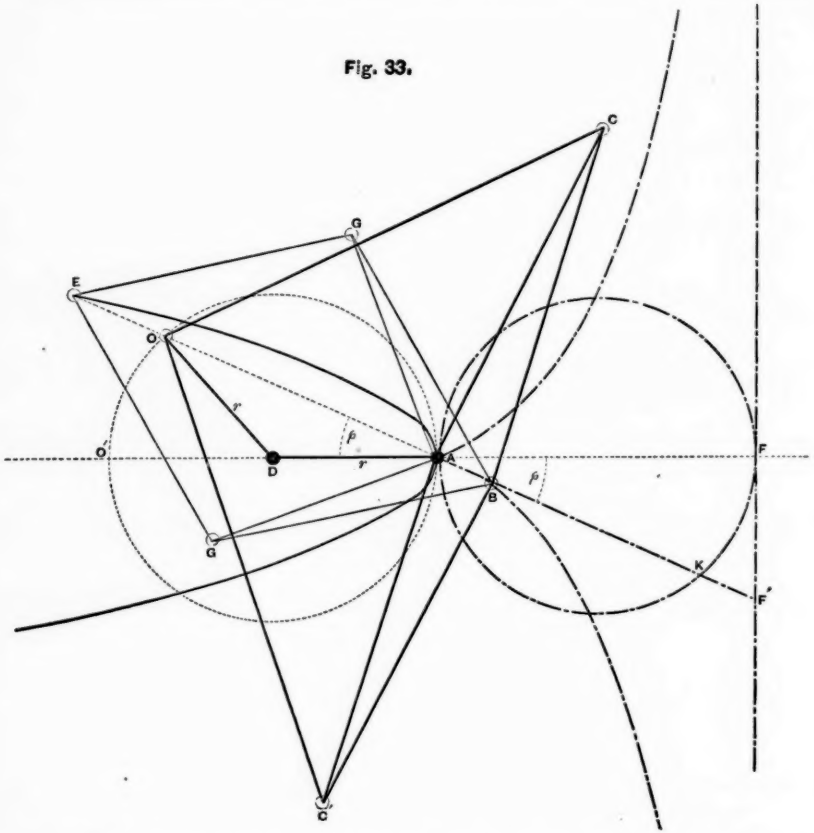
Calling the connectors OC and PB respectively m and m', we have by virtue of equation (2);

$$OA^2-AP^2=m^2-n^2 \quad \text{and}$$
$$AB^2-AC'^2=m'^2-n'^2,$$

or adding the two equations;

$$DA^2+AB^2=m^2+m'^2-2n^2+AP^2+AC'^2,$$

Fig. 33.



but since AB is always perpendicular to OA, we have;

$$AP^2+AC'^2=n^2,$$

and consequently;

$$OA^2+AB^2=m^2+m'^2-n^2.$$

Now, taking A as the fixed point, as indicated in Fig. 37, and making O move in a circle which passes through A, the radius of which is $AD=r$, then the point B describes a curve B'BR, which bears the name of *lemniscatoid*, and observing that;

$$AB^2=m^2+m'^2-n^2-OA^2$$

and $OA=2r\cos\varphi$,

it is clear that the equation of the curve is

$$\overline{AB}^2=m^2+m'^2-n^2-4r^2\cos^2\varphi,$$

or by substitution for $m^2+m'^2-n^2$ of the value of AB when $\varphi=0$

$$AB^2=(AB'^2-4r^2)-4r^2\cos^2\varphi,$$

while the substitution of the value of AB for $\varphi=90°$ gives,

$$AB^2=AR^2-4r\cos^2\varphi$$

When $AR^2=2r^2$ the lemniscatoid is transformed into the lemniscate of Bernouilli, and the equation becomes:

$$AB^2=-2r^2+4r^2\cos^2(90-\varphi)$$
$$=2r^2\cos(180-2\varphi)$$

and for $AR=2r$ the curve becomes two tangent circles whose equation is

$$AB=2r\sin\varphi.$$

Seeking the inverse value of the radius vector AB of the lemniscatoid, we shall have for the case of a real value of AB:

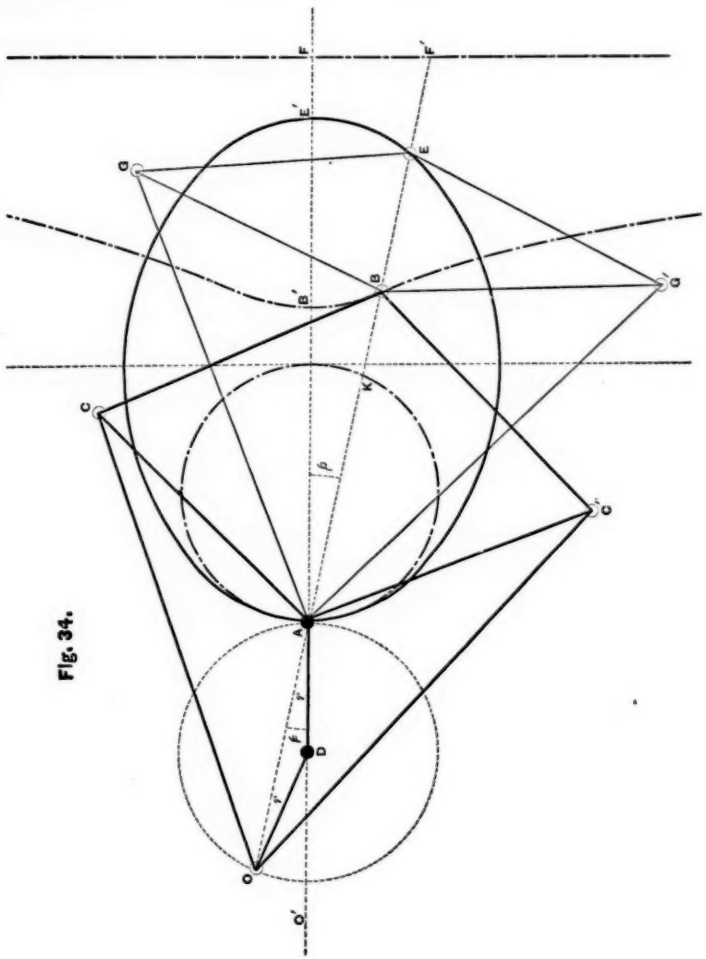


Fig. 34.

$$AE=\frac{C^2}{AB}=\frac{AB'.AE'}{\pm\sqrt{[AB'^2-4r^2-4r^2\cos^2\varphi]}}$$

or in taking

$$AE'=a \text{ and } AB'^2=\frac{4r^2b^2}{a^2-b^2}$$

$$AE=\frac{1}{\pm\sqrt{\left\{\frac{1}{b^2}-\frac{a^2-b^2}{a^2b^2}\cos^2\varphi\right\}}}$$

consequently the curve EE′ described by the point E is an ellipse whose axes are:

$$2a=2AE' \text{ and } 2b=\frac{2AE'.AB'}{\sqrt{4r^2+AB'^2}}$$

On the other hand, if AB′ is imaginary, we have, Fig. 38:

$$AE=\frac{C^2}{AB}=\frac{AR\,.\,AE}{\pm\sqrt{[AR^2-4r^2\cos^2\varphi]}}$$

or making

$$AE'=a \text{ and } AR=\frac{4r^2b^2}{a^2+b^2}$$

Fig. 35.

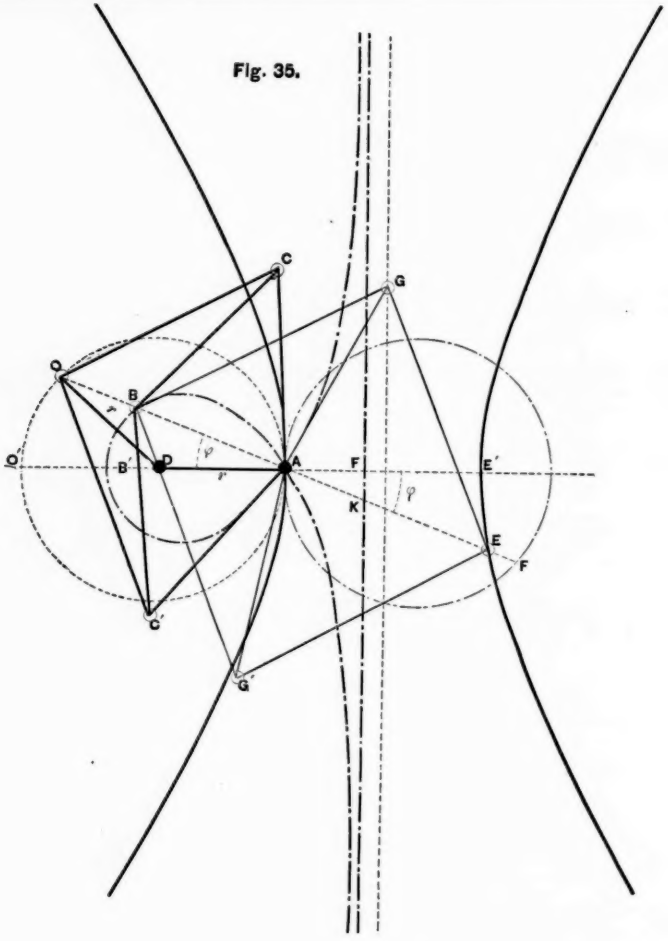


Fig. 36.

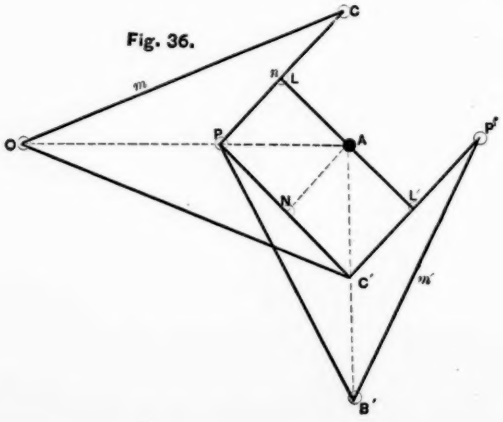


Fig. 37.

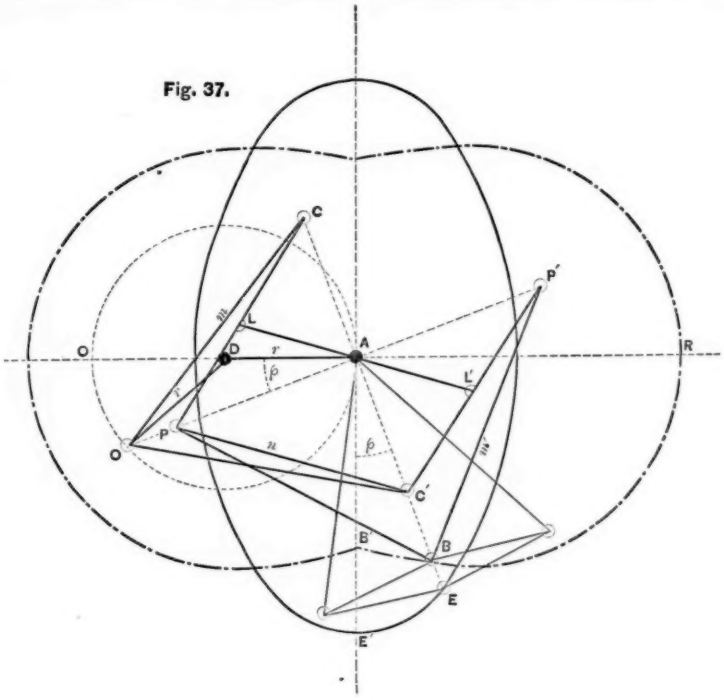


Fig. 38.

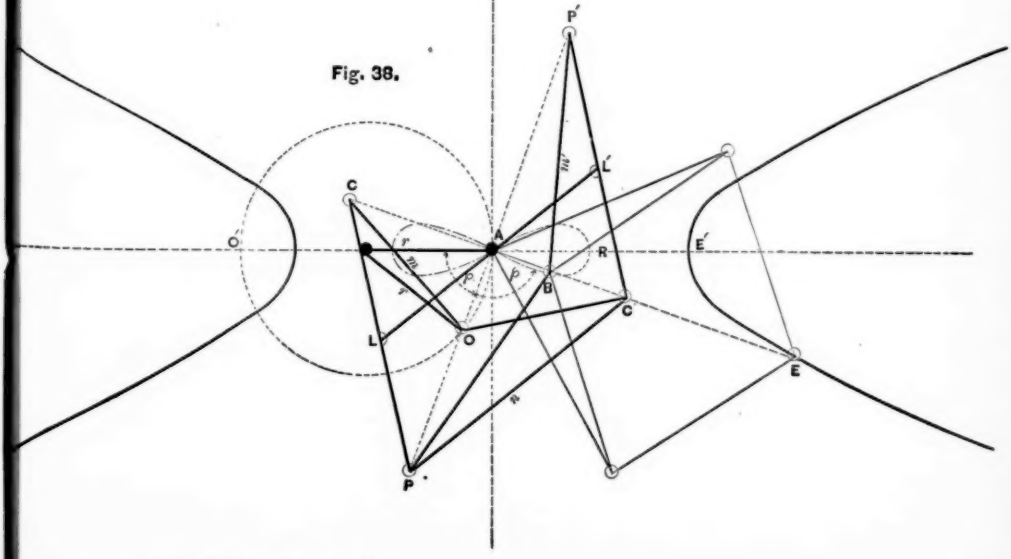
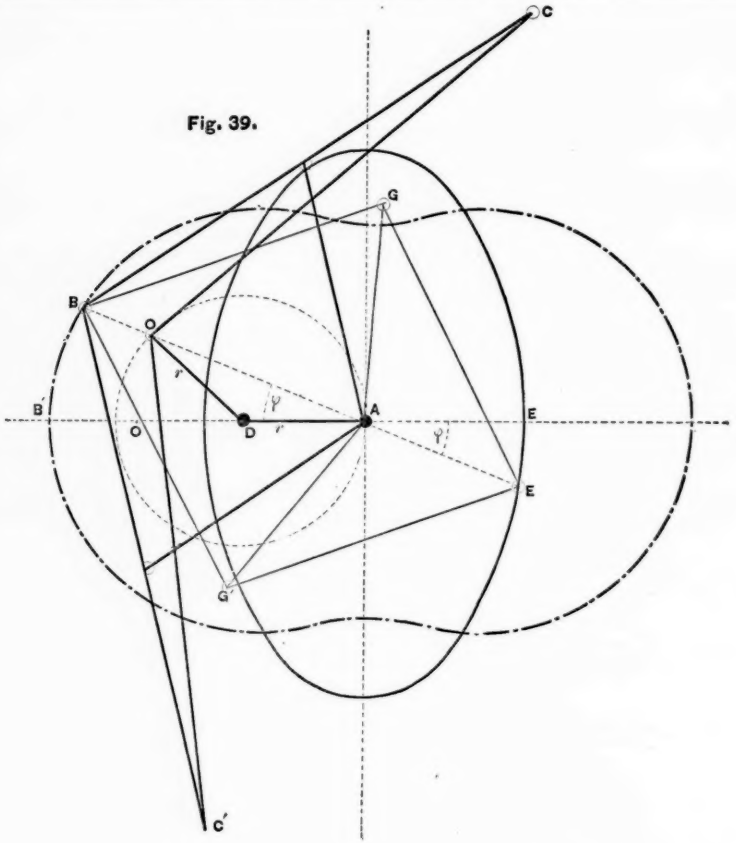


Fig. 39.



$$AE=\frac{1}{\pm\sqrt{\left\{\frac{1}{a^2}-\frac{a^2+b^2}{a^2b^2}\cos^2\varphi\right\}}}$$

and consequently the curve EE′ described by the point E is a hyperbola whose axes are:

$$2a=2AE' \text{ and } 2b=\frac{2AE'.AR}{\sqrt{4r^2-AR^2}}.$$

This tracing of the ellipse and hyperbola is due to Sylvester, and is especially remarkable for the simplicity of the transformation of cos φ to sin φ.

To appreciate this, it is necessary to recall the fact that it required in the case of Fig. 23, a combination of five elements, or thirty links, in order to pass from cos φ to sin φ, and the same combination was required to change

$$OA \text{ to } AK=\frac{C''^2}{C^2}\sqrt{C^2-OA^2}$$

while the same operation, that is to say, the change (Fig. 36) from

$$OA \text{ to } AB=\sqrt{C''^2-OA^2}$$

is here effected by a single element of eight bars.

In the first case, AK falls in the direction of OA, while in the second AB is perpendicular to OA.

But this system of eight links of Sylvester can be still further simplified, for the transformed cell of Peaucellier, containing only six bars, suffices to trace the conic sections around their center.

Uniting for this purpose, as shown in Fig. 39 and 40, the *element varié* CC′ to the ordinary element GG′, and de-

scribing from O a circumference which passes through the fixed point A, and of which the radius $AD=OD=r$, we shall have in this case, by virtue of eq. 2,

$$AB^2=C^2+OA^2$$

or supposing that B and O come to B′ and O′ when OD takes the direction AD,

$$AB'^2=C^2+O'A^2,$$

and further,

$O'A=2r$ and $OA=2r.\cos O'AO=2r\cos\varphi$

then $AB^2=AB'^2-4r^2+4r^2\cos^2\varphi$

or replacing φ by its complement φ'

$$AB^2=AB'^2-4r^2\cos^2\varphi'.$$

Consequently taking AB′, Figs. 39 and 40, equal to AR of Figs. 37 and 38, the curve described by the point B will be the same *lemniscatoid* in both cases.

The inverse value of AB is

$$AE=\frac{1}{\pm\sqrt{\frac{AB'^2-4r^2}{AE'^2.AB'^2}+\frac{4r^2}{AE'^2-AB'^2}\cos^2\varphi}}$$

and gives consequently a conic section and for $AB'>2r$ an ellipse (Fig. 39) whose axes are

$$2a=\frac{2AB'.AE'}{\sqrt{AB'^2-4r^2}}$$

and $2b=2AE'$,

and for $AB<2r$ an hyperbola (Fig. 40) having for axes

$$2a=2AE'$$

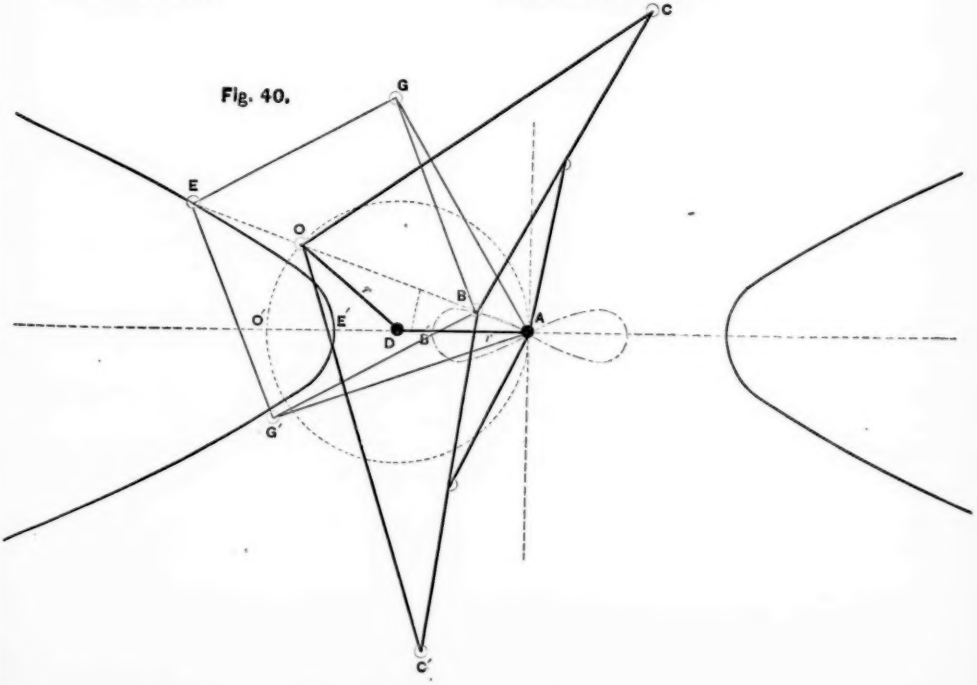
and $$2b=\frac{2AE'.AB'}{\sqrt{4r^2-AB'^2}}$$

This conicograph, the fourth which we have made known, improves not only in simplicity upon the third, but possesses the further advantage of describing with great facility the more flattened portion of the ellipse.

A conicograph more simple still is based upon the property that the curve described by ε, Fig. 26, under determinnate conditions is an inverse conic.

This curve is described by means of three links, by coupling the point B′ to the center O′ of the circular motion; but, as the deduction of the general formula requires very complex calculations, and as we have restricted this note mostly to the systems of six links, we will omit further reference to it.

Fig. 40.



Meanwhile we will present the calculation for the more important case in which OC=OO′, CB′—B′O′ and the point E is found on CB′.

Drawing in Fig. 41, where B′ has been replaced by B, the right line EN parallel to OC, EO″ parallel to the diagonal OB, and from the point O″ the intersection of EO″ and OO′ a right line O″M parallel to O′B; it is evident that the triangle is equal and similar to the triangle MOO″, and consequently,

$$NE=OO''.$$

And since, furthermore,

NE : OC :: BE : BC and OC, BE and BC, which we will designate respectively by p, w and q are of invariable length, it is necessary that

$$OO''=NE=\frac{pw}{q}.$$

The length OO″ is therefore equally invariable, and we may choose O″ as the origin of a system of polar co-ordinates.

Taking O″E=z; angle CBO=OBO′ =β; angle COB=BOO′=EO″O=φ; angle XCY=2φ, and XCZ=$\beta-\varphi$, and since

$$VE=BU+WB$$

$$z\sin\varphi=q\sin(\beta+\varphi)+w\sin(\beta-\varphi)$$
$$=q\sin\beta\cos\varphi+q\sin\varphi\cos\beta+w\sin\beta\cos\varphi-w\sin\varphi\cos\beta$$
$$=(q+w)\sin\beta\cos\varphi+(q-w)\sin\varphi\cos\beta.$$

Furthermore we have in the same figure

$$p.\sin\varphi=q\sin\beta$$

or
$$\sin\beta=\frac{p}{q}\sin\varphi, \text{ and}$$

$$\cos\beta=\pm\sqrt{1-\frac{p^2}{q^2}\sin^2\varphi}$$

so that the value of z becomes after substitution of $\sin\beta$ and $\cos\beta$;

$$z=\frac{p}{q}(q+w)\cos\varphi\pm$$
$$\sqrt{(q-w)^2-\frac{p^2}{q^2}(q-w)^2\sin^2\varphi}$$

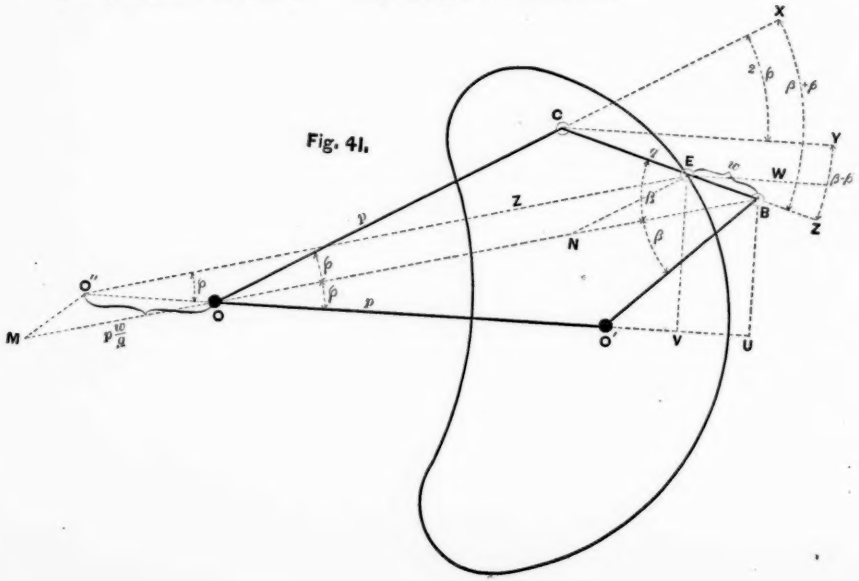
an equation of which both signs may be satisfied.

The curve ET(Fig. 41) which this figure represents, can be obtained by a point on a curve, which rolls upon another curve, in the same manner as a point in a circumference which rolls on a line generates the cycloid.

M. Mannheim has made a study of these curves to which Reuleaux has given the general names of *polar curves* (*courbes ou lignes polaire*) (*polbahnen*).

He has found that the displacement of CB relative to OO″, and consequently the curve described by E is produced by an oval of Descartes rolling upon an oval of Descartes.

Fig. 41.



REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The Board of Direction submit the following list of topics, on which original papers, illustrating the experience of the writers, are requested:

1. Topographical Surveys, and the laying out of Towns and the most economical divisions of property.
2. Instruments and methods of exact measurement for land surveys.
3. Trigonometrical and Astronomical surveys.
4. Systems of roadmaking and maintenance suitable for (*a*) Large towns and heavy traffic; (*b*) Suburban districts; (*c*) Rural districts.
5. The sustaining power of different kinds of soils and the conditions which produce failure of foundations by settlement.
6. The weight which can be supported by different classes of masonry.
7. The preservation of masonry structures.
8. The manufacture of cements and methods of testing their strength.
9. The proportions of cement and inert materials in mortars.
10. The preservation of timber used in construction.
11. The strength, durability and characteristics of various timbers.
12. The manufacture of iron and steel in America. Combinations of materials, methods employed and plant used.
13. The properties and laws of cast iron, wrought iron, steel and other metals used in construction.
14. The effect of constant or long continued vibration on metals used in construction.
15. The design, generally, of iron bridges of large span.
16. The construction and operation of drawbridges of large span.
17. Testing machines and experiments on the strength of materials and structures.
18. Appliances and methods of rock boring and blasting.
19. Sub-aqueous tunnels. Their design and construction.
20. The flow of water in channels of various kinds, and the modes of determining the discharge by experiment. Also the discharge over weirs and through orifices.
21. The loss of water in flowing in open channels.
22. The relations of rainfall, character of soil, and flow of streams; effect of wooded and of cleared and cultivated land on the flow of streams.
23. The sources and systems of water supply for towns, including storage of surface water, open wells, tube wells, gathering galleries, and ground water obtained from different geological strata.
24. Systems of water supply suitable for small communities, whether separately or co-operatively.
25. The detection and prevention of waste of water in towns.
26. The design, construction, and cost of operation of pumping engines, and mode of computing duty.
27. The history of the manufacture of cast iron water pipes in America.
28. The strength and dimensions of cast iron pipe and of wrought iron pipe for water supply. Forms of joints for water pipe.
29. Durability and cost of water pipe of cast iron, wrought iron and cement, wood, and other materials.
30. The design and construction of earth embankments and masonry dams for reservoirs.
31. The constructions, dimensions, and operation of navigable canals.
32. Methods of overcoming differences of level in canals.
33. The improvement and training of rivers.
34. The construction of harbors of refuge.
35. The construction of wharves, piers and docks.
36. The construction of fire-proof buildings.
37. The warming and ventilation of buildings.
38. House drainage.
39. The sewerage of towns and the disposal of sewage.
40. The form, material and dimensions of sewers.
41. The economical location of railroad lines.
42. The construction and maintenance of the permanent way on railroads.
43. The form, manufacture and life of rails.
44. The form and material of railway wheels.
45. The heating and lighting of railroad cars.
46. The prevention of accidents at railroad crossings and intersections.
47. Rapid transit in large cities.
48. The prevention of noise from heavy railroad trains moving at high speed.
49. Safe substitutes for animal power on surface railways.
50. The relative economy of the several kinds of fuel used in locomotive engines.
51. The operation of freight and passenger traffic on railroads, and the conditions of economy in the same.
52. The arrangement of terminal stations on trunk lines of railroads.
53. The manufacture, distribution, measurement and use of illuminating gas.
54. Electricity as applied to lighting and to motive power.
55. The modern construction of water wheels and engines.
56. The engineering questions involved in the location and management of large Industrial Exhibitions.
57. Manufacture, strength and durability of earthenware pipe.
58. The relations of Engineers to Employers and to contractors.
59. The Status and Compensation of Experts.
60. Engineering and Mechanical Law.

IRON AND STEEL NOTES.

THE GROWTH OF THE NORTHERN STEEL TRADE.—Although the great declension in the iron manufacturing trade of the north is now well known, it is far from so fully known that there is to some extent a substitutional

growth of the steel manufacture. Three or four years ago the steel trade was confined to the Newburn Works on the Tyne, and the small works at Tudhoe, and to the still smaller places at Guisbrough—the first and last using the old processes, the second that of Bessemer, but on a small scale, the capacity of the two converters being 2 tons 10 cwt. each. Since that time the large works at Eston have been constructed, the Siemens process has been introduced at Newburn, and at Darlington, Middlesbrough, and Geisbrough there are now in progress extensions of the Bessemer system, whilst a Siemens plant is nearly ready for use at Middlesbrough. Whereas, at the earlier date mentioned, the productive capacity of the Bessemer plant in the north was that of converters of a total capacity of 5 tons, there are now at Eston and Tudhoe converters the aggregate capacity of which is 37 tons, and to this the additions already indicated must shortly be made. By these additions, accomplished and in progress, the production of steel in the North of England will be raised from a few thousand tons yearly to about a quarter of a million—the Eston Works alone having a capacity of production of at least one hundred thousand tons yearly. With abundant supplies of ore now proved available for use in the steel manufacture, with admirable smelting plant, and with full fuel supplies, and excellent facilities of transport of both materials and finished product, there is a probability of even further extensions of the steel trade in the North of England at the expense of the iron manufacture. There is now a certainty of a larger use of steel in the local shipyards, and with that there will of necessity arise a development of the steel angle and plate manufacture, which has hitherto been very slightly provided for in the extension and construction of steel works in the north.

RAILWAY NOTES.

FAST TRAINS ON ENGLISH RAILROADS.—A recent number of the London *Railway News* has the following:

Among other improvements which have been effected by the prudent application of capital to duplicating lines, putting in new sidings for marshaling traffic, is the greater speed of the trains generally, and corresponding saving in fuel, wages, etc. The subject has been duly noticed at the half-yearly meetings. On the other hand, however, there is a seeming disposition to drive competitors out of the field by means of quickest time, and this has led to instances of rapid traveling which may operate in a contrary direction in the matter of working cost. Particulars as to the speed now made upon the principal lines will, perhaps, be of some interest.

The Great Eastern chairman pointed out to the proprietors at the last half-yearly meeting that, by the duplication and remodeling of their lines, they were now able to run trains from Norwich to London in 3 hours, as compared with about 4½ hours. The saving of time and working expense in this case is very considerable. The continental express is now run from London to Harwich at the rate of 38½ miles an hour. They run at much higher speed, however, on some sections of the line. Between London and Chelmsford a speed of 42 miles is attained, while between Bishop's Stortford and Cambridge, and between London and Colchester, they make as much as 44 miles an hour, this being the highest average speed they record.

Numerous express trains on the Great Northern are timed to run from London to Peterborough in 1½ hours, the distance being 76½ miles, and the average 51 miles an hour. Taking into account the number of trains which make this high average, and the almost uniform record upon all the main lines of the company, the Great Northern undoubtedly occupy the foremost ranks in the matter of speed. The Scotch express covers 105 miles in 2 hrs. 8 min. without stoppage, which is probably not only the longest continuous run, but likewise the fastest, upon the standard gauge, to be found anywhere in the world.

The Great Western Company's express trains get over the 36 miles between London and Reading in 46 min. at the rate of 47 miles an hour. An exceptional rate of speed has been given to this company's Exeter express trains so as to put them on a time level with the Southwestern, which have a much shorter route to that city. These roads travel over the broad gauge from London to Swindon in 1 hr. 27 min , being the extraordinary equivalent of 53½ miles an hour. Upon the announcement, a short time ago, that an additional fast afternoon train would be run by this company, the London & Southwestern appear to have reduced their time by way of retaliation, and we shall probably soon see a similar reduction in the Great Western. To assimilate the times it will be necessaay to increase the speed by about 4 miles an hour.

The Brighton Company have trains which run from London to Brighton, the distance being 50½ miles, without stopping, and the average speed 43½ miles an hour. Others accomplish that part of the journey between Croydon and Brighton at the rate of 46 miles, and the Portsmouth express trains on some parts of their journey average as much as 47½ miles an hour, showing that where there is competition the speed is greatest.

The London, Chatham & Dover Company's continental express trains, and the well-known "Granville" express train on the same line, average 45 miles an hour.

The best average runs are made on the London & Northwestern, between London and Rugby, where a speed of 45 miles is made by a considerable number of trains, and one Manchester train performs that part of the journey between Willesden and Rugby at the rate of 47½ miles an hour, this being the maximum velocity for any continuous journey by this company's trains.

The London & Southwestern is a comparatively slow line. Their fastest time to Portsmouth is 2 hrs. 10 min., the distance 73½ miles; to Southampton, 2 hrs. 20 min., and the distance 79 miles. The run from London to Woking is made at

the rate of 41½ miles an hour. Their West of England express train is, however, the fastest. This train having been recently accelerated, now traverses the distance between London and Exeter, 171½ miles, in four hours, a fairly creditable performance. The best time is made in the run from London to Basingstoke, at the rate of 44 miles an hour.

The Midland have a comparatively easy run between London and Bedford, a distance of 48 miles, which is done by the morning "newspaper" train in just three minutes over the hour, the actual rate of speed being therefore 46 miles an hour. The severe grades upon the Settle & Carlisle line do not prevent this company from maintaining almost the maximum upon the long journey of 86½ miles between Skipton and Carlisle. This difficult journey is accomplished in five minutes under the two hours, equivalent, therefore, to 45 miles an hour. Upon this and the corresponding section of the London & Northwestern from Preston to Carlisle the nature of the country has necessitated a succession of long and severe gradients, and the important trains, before entering upon it, are usually divided and proceed in two portions. On some of the long downhill stretches, a speed of 70 miles an hour is occasionally obtained.

The Southeastern Company's continental trains run from Cannon street to Dover, a distance of 75 miles, in 1 hr. 42 min., being at the rate of 45 miles an hour.

The recent brake trials disclosed some instances of extreme velocity, the experiments taking place at speeds up to 60 miles an hour.

The chairman of the London & Northwestern two years ago mentioned that his company had refused to go into the severe competition in point of speed, and that advantage was sought more by the geography of the line, and that their trains worked at a less speed than any of the other companies. The following summary of the maximum averages now made on the different lines will show how different the views of some of the companies appear to be upon this important matter:

	Miles.
Great Western	53½
Great Northern	51
London & Brighton	47½
London & Northwestern	47½
Midland	46
London, Chatham & Dover	45
Southeastern	45
Great Eastern	44
London & Southwestern	44

—*R. R. Gazette.*

ENGINEERING STRUCTURES.

THE LONGEST RAILWAY BRIDGE ON THE CONTINENT.—According to the *Weser Zeitung*, the bridge over the Rhine at Wesel comprises 107 arches or openings. Of these there are four in the center of the stream 313 feet wide, the less considerable including 35 of 60 feet each, 6 of 61 feet, 14 of 40 feet, 18 of 12 feet, 10 of 20 feet, and 20 of 30 feet. The total length of the bridge or viaduct, including the part on piles, is, therefore, 6,220 feet, or 1,952 meters.

ELEVATION OF RAILWAYS.—The elevation above the level of the sea of some of the chief railways of the world, out of the British Islands, is as follows:—The Apennine line at its highest point is 617 metres above the sea level, the Black Forest line 850 metres, the Semmering 890, and the railway over the Caucasus 975 meters. The tunnel of the St. Gothard line attains an elevation of 1,154 meters, the Bremner line 1,367, and the Mont Cenis line 1,338. In America the highest lines are the North Pacific, which at its most elevated point is 1,652 meters above the level of the ocean, the Central Pacific 2,140, and the Union Pacific, 2,513 meters. The highest point of the railway over the Andes is 4,769 metres above the sea level.

A PRUSSIAN RAILWAY BRIDGE.—One of the longest railway bridges in Germany is now on the point of completion. It crosses the river Vistula, near Grandenz, and rests upon twelve piers. The length from the commencement of the first to the end of the last pier is 1,443 meters, or nearly a mile. It is considerably longer than the other chief bridges over the Vistula, at Dirschau and Thorn, and is likewise lighter and more elegant than the Dirschau bridge, though the cost of its construction was not half as much. It will connect the East Prussian with the Thorn and Insterburg Railways, and bring the province of West Prussia into direct connection with the main lines of international traffic.

ORDNANCE AND NAVAL.

THE relative cost of rival great guns is an important factor in the gunnery problem, and is as follows: The Krupp steel gun of 70 tons, £22,000; the Armstrong 100 ton coiled wrought iron gun £16,000; the Woolwich—Fraser—wrought-iron coil gun, 80 tons, £10,000. Therefore, two 80-ton guns of the Woolwich type can be constructed for the cost of one of their German rivals, and leave £2,000 to the good.

THE ARMSTRONG DIVIDED MOUNTAIN GUNS IN AFGHANISTAN.—The interest in the operations in Afghanistan center in the movement of the column advancing from the Kurram Valley over the Shatargardan Pass on Cabul, under Sir Frederick Roberts. This column having secured the road over the above-mentioned pass, has about seventy miles to traverse of bad but feasible ground with but one fairly difficult pass in its path. It is satisfactory to be able to state that it is well provided with suitable artillery in proportion to its strength, including the two mountain batteries of experimental Armstrong guns of the pattern described in the last volume of the *Engineer*. These guns having been made on a suggestion of Col. Le Mesurier to take asunder, are so constructed as to be really powerful field pieces while having the mobility of mountain guns, each gun being carried on two mules. It may be remembered that we pointed out that even should experience teach us that the union of

the parts of the barrel was a more difficult process in action than the putting together of a mountain carriage, the battery would still be specially suited to such operations as involved the crossing of a range of mountains and subsequent fighting on level ground. This is precisely the condition under which these guns are now to be employed. While the daily papers may with truth observe that we may well congratulate ourselves on the presence of officers of such reputation as Sir F. Roberts, Brigadier-General Massey and others who happen to be within reach at the moment, we are glad to be able to point out that these two experimental batteries—twelve guns in all—are exactly in the right place at the right time. We hope to hear of good results effected by them.

BOOK NOTICES

NAVIGATION AND PRACTICAL ASTRONOMY. By REV. W. T. READ, M. A. London: Elliot Stock For sale by D. Van Nostrand. Price, $4.25.

A book for learners who have progressed as far as trigonometry in their mathematics. Numerous examples worked out, illustrate the methods of calculation.

The difficulties in navigating iron vessels are specially treated.

A MANUAL OF SIGNALS. FOR THE USE OF SIGNAL OFFICERS IN THE FIELD, AND FOR MILITARY AND NAVAL STUDENTS. By BV'T. BRIG. GEN. ALBERT J. MYER. Washington: Gov't Printing Office.

This is a new and improved edition of a book that has been in use for some years. It is preeminently an instruction book; the fullest details being afforded in the way of directing the learner, who is desirous of acquiring proficiency in signalling.

THE ELEMENTS OF MODERN TACTICS PRACTICALLY APPLIED TO ENGLISH FORMATIONS. By WILKINSON J. SHAW, M. D. London: C. Keegan Paul & Co. For sale by D. Van Nostrand. Price, $4.00.

This is one of a series of "Military Hand-Books for Officers and Non-Commissioned Officers." It is of convenient size, compactly printed and fully illustrated; the colored lithographic maps being especially noticeable.

TREATISE ON THE CONSTRUCTION AND MANUFACTURE OF ORDNANCE IN THE BRITISH SERVICE. For sale by D. Van Nostrand. Price, $4.50.

This work contains the full description of the guns of the British Army and Navy at this present date.

An important part of the work is the treatise on "Gun construction generally." Quite a large portion is devoted to tables, relating to the use of guns of different sizes.

TEXT-BOOK OF FORTIFICATION AND MILITARY ENGINEERING, FOR USE AT ROYAL MILITARY ACADEMY AT WOOLWICH. London. For sale by D. Van Nostrand. Price, $14.25.

Two volumes, royal octavo, with many illustrations, both colored and plain. No detail of construction, that has been considered worthy of mention in the text, seems to be without its graphic representative among the cuts. A portion of the work is in a sense historical, representing operations in famous sieges, and is thus far at least interesting to the peaceful citizen. But the book is of great value only to those whose occupation is scientific warfare.

THE GRAMMAR OF LITHOGRAPHY. A PRACTICAL GUIDE FOR THE ARTIST AND PRINTER. By W. D. RICHMOND. London: Wyman & Sons. For sale by D. Van Nostrand. Price, $2.50.

The practical and technical character of this book is unquestionable. It comprehends the details of Commercial and Artistic Lithography —Chromo Lithography, Zincography, Photolithography and Lithographic Machine Printing.

The illustrations are not numerous, but probably will suffice for the artisan who is already skilled in the general manipulation; and it is for such that the work is manifestly intended.

BOOTS AND SADDLES. A HISTORY OF THE FIRST VOLUNTEER CAVALRY OF THE WAR. By JAS. H. STEVENSON. Harrisburg: Patriot Pub. Co. For sale by D. Van Nostrand. Price, $1.50.

This Regiment was known as the First New York (Lincoln) Cavalry.

The book contains a spirited narrative of the part taken by this troop during the war. The illustrations are so poor that the appearance of the book would be improved by their omission. The defects are largely due to the artist, but it cannot be denied that he was much aided by the engraver.

The story will doubtless prove interesting to that large audience who actively participated in the other scenes of the great drama of the war.

ICE-MAKING MACHINES. SCIENCE SERIES, No. 46. FROM THE FRENCH OF M. LEDOUX. New York: D. Van Nostrand. Price 50 cts.

In this brief treatise, as the readers of the Magazine already know, the various ice machines are regarded as being of three different types, represented by—the Air Machine of M. Gifford; the Sulphurous Acid Machine of M. Pictet, and the Ammonia Machine of M. Carré.

The theoretical conditions of successful working of each type are elaborately presented, and enough of the practical details is given to explain the rationale of the operation.

As an application of the principles of thermodynamics, the treatise will prove of interest to the student, while, as an exposition of the relative merits of the different processes, it will prove of undoubted value to the practical engineer, who is called upon to employ a refrigerating machine for any of its several uses.

WAVE AND VORTEX MOTION. By THOMAS CRAIG, PH. D. New York: D. Van Nostrand. Price 50 cts.

The difficult work successfully accomplished by engineers, and the marvelous progress made by inventors during the past half-century, may

be almost entirely traced to the greater attention paid to secure accuracy and to exercise judgment in the application of natural laws. The man who now depends upon chance experiments for giving him desired results is soon left far behind by him who avails himself of the knowledge which has been obtained, by systematic scientific investigation: and, as it is not to be expected that the proceedings of the various learned societies, nor even the various technical periodicals, can be regularly consulted, such treatises as those contained in Van Nostrand's Science Series are invaluable. The last issued volume—No. 43—is in no respect inferior to any of its predecessors, whilst it deals with a subject which will be of great utility to a large number of practical men. It contains the elements of the mathematical theory of fluid motion in a form which will enable those who study the memoir to grasp the principal facts easily and thoroughly. That problems in hydrodynamics are frequently very difficult cannot be doubted, and this difficulty is not diminished by the fact that no general work upon the subject exists, so that the student has to pick up his information as best he can. This obstacle to progress Dr. Craig proposes to remove by treating the entire subject systematically and exhaustively, and the present Science Series volume (Elements of the Mathematical Theory of Fluid Motion—Wave and Vortex Motion. By Thomas Craig, Ph.D., Fellow in Physics in the Johns Hopkins University, Baltimore. New York: Van Nostrand. London: Trübner & Co.) furnishes an outline of and introduction to the complete treatise which he intends shortly to publish. Dr. Craig has taken especial care that the students shall be led on by easy steps to the thorough comprehension of the subject. Commencing with the consideration of the general equations of fluid motion, he demonstrates that the circulation in any closed line moving with the fluid remains constant. He then explains such leading principles of the theory of the potential as frequently recur, and explains the derivation of them; examines the motion of water in plane waves when the excursions of each particle are very small; discusses the generation of cylindrical waves, and then considers free vortex motion generally. The style of treatment is clear and explicit, and will be thoroughly intelligible to anyone who has made but limited progress in mathematical study. Dr. Craig's complete treatise will be looked forward to with much interest.—*London Mining Journal.*

FUEL. ITS CONBUSTION AND ECONOMY. By D. KINNEAR CLARK, C. E. New York: D. Van Nostrand. London: Crosby Lockwood & Co. Price, $1.50.

In this practical treatise the reader is first presented with careful abridgements of two standard works; the first "on the Combustion of Coal and the Prevention of Smoke," by C. Wye Williams, A. I. C. E., and the second "on Economy of Fuel," by T. Symes Prideaux. In the remaining portion of the work the Editor has summarised the more recent results of progress in the combustion, and economical use, of fuels on the lines laid down in the treatises above mentioned, giving also considerable space to an important series of experiments with coal under steam boilers, conducted separately by Mr. Longridge at Newcastle-on-Tyne, and by Mr. Fletcher at Wigan.

Gas furnaces are treated at considerable length, in view of their great promise for the future.

The table of contents given below will best exhibit the comprehensive character of the book. It should however be added, that the illustrations number 144, and are interspersed in the text.

Part I.—On the Combustion of Coal, and the Prevention of Smoke.

BY C. WYE WILLIAMS, A.I.C.E.

PPEFACE TO THE FIRST EDITION:—Chapter I, On the Constituents of Coal, and the Generation of Coal-Gas; II, Of Gaseous Combinations, and particularly of the Union of Coal-Gas and Air; III, Of the Quantity of Air Required for the Combustion of Carbon, after the Gas has been Generated; IV, Of the Mixing and Incorporation of Air and Coal-Gas; V, Of the Principles on which Boilers and their Furnaces should be Constructed; VI, Of the Introduction of the Air to the Fuel, in a Furnace, Practically Considered; VII, Of Regulating the Supply of Air to the Gas by Self-Acting or other Mechanical Apparatus; VIII, Of the Place most Suitable for Introducing the Air to the Gas in a Furnace; IX, Of Various Furnace Arrangements, with Observations thereon; X, On Providing Adequate Internal Surface for Transmitting the Heat to the Water for Evaporation; XI, Of Flame, and the Temperature Required for its Production and Continuance, and its Management in the Furnaces and Flues; XII, Of the Circulation of Water in the Boiler; XIII, On the Circulation of the Water in Relation to Evaporaiion, and its Influence on the Transmission of Heat; XIV, Of the Circulation of the Water in Relation to the Durability of the Plates; XV, Of the Draught; XVI, Of the Tubular System as Applied to Marine, Land, and Locomotive Boilers, in Reference to the Circulation of the Water and the Process of Combustion; XVII, On the Use of Heated Air, and its Supposed Value in the Furnaces of Boilers; XVIII, On the Influence of the Water Generated in Furnaces from the Combustion of the Hydrogen of the Gas; XIX, On Increasing the Heat-Transmitting Power of the Interior Plate-surface of Boilers; XX, On the Generation and Characteristics of Smoke; XXI, Concluding Remarks.

APPENDIX.—Extracts from the Second Report of Messrs. Longridge, Armstrong, and Richardson to the Steam Coal Collieries' Association, Newcastle-on-Tyne.

Part II.—On Economy of Fuel.

BY T. SYMES PRIDEAUX.

INTRODUCTION.—Chapter I, On the Best Means of Rendering Combustion Perfect; II, On Contrivances for the Employment of Inferior Kinds of Fuel; III, On the Use of Compressed Air in Reverbatory Furnaces; IV, On the Economy to be Attained by Increasing the

Temperature of Furnaces; V, On Feeding Furnaces with Hot Air; VI, On the Manufacture of Iron.

Part III.—Fuels: Their Combustion and Economical Use.

BY D. K. CLARK, M. INST. C.E.

Chapter I, Chemical Composition of Fuels, and Formulas for Combustion; II, Coal; III, Combustion of Coal; IV, Evaporating Performance of Coal in a Marine Boiler at Newcastle-on-Tyne; V, Evaporative Performance of Coal in Lancashire and Galloway Boilers at Wigan; VI, Coal-Burning in Locomotives; VII, Coke; VIII, Lignite, Asphalte and Wood; IX, Peat; X, Tan, Straw, and Cotton-Stalks; XI, Liquid Fuel—Petroleum; XII, Total Heat of Combustion of Fuels; XIII, Gas-Furnace:—Function and Operation of Gas-Furnaces; XIV, Application of Gas-Furnaces for the Manufacture of Gas; XV, Gas-Furnaces from Steam-Boilers; XVI, Decomposition of Fuel in Gazogenes; XVII, Iron-Furnaces. — Ordinary Furnaces; XVIII, Utilizing the Waste Heat of Ordinary Iron-Furnaces by Generating Steam; XIX, Iron-Furnaces in which Waste Heat is Utilized by Heating the Air; XX, Blast-Furnaces; XXI, The Siemens Regenerative Gas-Furnace; XXII, The Ponsard Gas-Furnace, with Recuperator; XXIII, Gorman's Heat-Restoring Gas-Furnace; XXIV, Water-Gas Generators for Heating Purposes; XXV, Powdered Fuel.

BOOKS RECEIVED.—List of Periodical Engineering Literature published in the English language. By Herman Houpt, Jr., Ph. D., Philadelphia.

Report of the Department of Public Works of the City of New York, for the quarter ending June 30th, 1879, with a special report on the subject of the water supply.

Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac. Vol. I., Part 1., Eclipses. Printed by authority of the Honorable, the Secretary of the Navy. Washington: Government Printing Office.

Methods and Results. Secular change of magnetic declination in the United States and at some foreign stations. Third edition with two plates. Washington: Government Printing Office.

The Acta Pilati. The official report of Pontius Pilate to the Emperor Tiberias, concerning the crucifixion of Christ. Edited by Rev. Geo. Sluter, A. M. Shelbyville, Ind.: M. B. Robins.

MISCELLANEOUS.

ON the occasion of the inauguration of M. Thiers' statue, on August 4th, an aëronautical ascent was made at Nancy. The wind was blowing from the west with a velocity of 16½ kilometers an hour, and no variation in the direction was perceptible from the ground to 800 meters. The altitude of the balloon was taken by officers from Mazléville, with a theodolite, and signals were exchanged with the ground during the ascent. The signals were given by the aëronauts with a flag, and by officers with a reflecting mirror placed in the end of a tube, and mounted as a telescope. The officers directed the rays of the sun on the balloon, and intercepted rays with a key in order to use the Morse alphabet. The distance of Mazléville from the balloon was more than 6 kilometers, and the signals, *Nature* says, could be seen at a much greater distance. This shows that in a besieged town a passing balloon could be used for giving orders to, or receiving news from friendly forces. The system of communication has been invented by one of the officers of the garrison. M. W. De Vonveille was in the car.

THE *American Manufacturer* says that "an undeveloped yet promising market for farming implements is reported in Morocco by U. S. Vice-Consul John Cobb, at Casablanca. In a recent communication, that officer, who takes a lively interest in the promotion of American trade, writes that farming implements are much needed in that country, no improvements having been made there in that line since the days of Mohammed the Great, nearly 1,300 years ago. Mr. Cobb believes our manufacturers will find a large field for operations there, as many of the Moors have money and are particularly fond of useful inventions. They are very conservative, however, and must see an article in use or under conditions in which it can undergo a thorough investigation before they can be made to believe in it. American goods are favorably received by them, and can be made to take the lead. Possibly our manufacturers interested in the export trade may find it worth while to correspond with Mr. Cobb." Perhaps some of our manufacturers can write to some one else, or go to see into the matter for themselves.

M. FERDINAND DE LESSEPS has issued the prospectus of his new scheme for piercing the American isthmus. The company to be organized for this purpose is to be called the "Inter-Oceanic Canal Universal Company," and its capital is to be 400,000,000f., or £16,000,000 nominal. This capital is to be divided into 800,000 shares of 500f. or £20 each, and 790,000 of these will be offered to the public, 10,000 being reserved for the original concessionaries in payment of concessions and surveys transferred by them to M. de Lesseps. Only 125f. per share will be called up in the first instance, the balance being taken as required, and interest at the rate of 5 per cent. will be paid on the actual money received during the course of the construction. Subscriptions will be opened in Europe and America on the 6th and 7th of August next. In his memorandum prefixed to the prospectus M. de Lesseps estimates an income of 90,000,000f. from the canal when it is completed, and as 85 per cent. of the profits are assigned to the shareholders he reckons that they will get 47,000,000f. per annum, or 11½ per cent. We have already shown we believe conclusively, that any such hopes of profit as those held out by the above prospectus must be delusive.

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A PRACTICAL TELEGRAPHER.

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THE AUTHOR.

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